The Effects of Some Thermo-physical Properties of Fluid on Heat and Mass Transfer Flow Past Semi-infinite Moving Vertical Plate with Viscous Dissipation

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Authors' contributions

This work was carried out in collaboration among all authors. Author MOD designed the study. Author KAJ performed the numerical analysis and wrote the first draft of the manuscript. All three authors managed the analyses of the study. Authors MOD and KAJ managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

This paper examines the effect of some thermo-physical properties of fluid on heat and mass transfer flow past semi-infinite moving vertical plate. The fluid considered is optically thin such that the thermal radiative heat loss on the fluid is modeled using Rosseland approximation.The governing partial differential equations in dimensionless forms are solved numerically using the Method of Lines (MOL). The velocity, the temperature, and the concentration profiles of the flow are discussed numerically and presented. Numerical values of the skin-friction coefficient, Nusselt number, and Sherwood number at the plate are discussed numerically for various values of thermo-physical parameters and they are presented by the tables.The result shows that an increase in thermal radiation causes increase in velocity and temperature profiles of the flow, thus,

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the thermal radiation intensifies the convective flow. Also, an increase in Soret number causes increase in velocity and concentration profiles of the flow while the effect is negligible on temperature profile distribution. Similarly, an increase in Dufour number causes increase in velocity and temperature profiles of the flow.

1. INTRODUCTION

The consideration of natural convection induced by buoyancy forces from thermal and mass diffusion is of great interest in view of its applications to geophysics, drying process etc., Alao, et al. [1] and many engineering problems such as cooling of nuclear reactors, astrophysics, boundary layer control in aerodynamics and cooling tower under the influence of magnetic field, Joytsana, et al. [2].

Prasad and Reddy [3] have studied the radiation and mass transfer effects on an unsteady MHD convective flow past a heated vertical plate in a porous medium with viscous dissipation. Ferdows, et al. [4] studied Soret and Dufour effects on natural convection heat and mass transfer flow in a porous medium considering internal heat generation. While analyzing the heat and mass transfer characteristic of flow using exponential form of internal heat generation, they suggested that the velocity, temperature and concentration flow fields are appreciably influenced by Dufour and Soret effects. Furthermore, in their analysis, with increasing Dufour number and decreasing Soret number, the velocity and concentration distributions reduced significantly, while temperature distribution increased along the flow field. Motsa and Shateyi [5] studied the effects of Soret and Dufour on steady MHD natural convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation in the presence of a chemical reaction. In the analysis of the model, they remarked that an increase in Soret and Dufor parameters increases significantly the velocity and concentration profiles of the flow but noted that Dufour effect enhances flow velocity much more than Soret. In many chemical engineering processes, there occurred chemical reaction between a foreign mass and the fluid in which the plate is moving. Rajesh and Vijaya [6] investigated radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Gnaneswara and Bhaskar,

[7] studied the effects of Soret and Dufour on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Vempati and Laxmi-Narayana-Gari [8] investigated the effects of Soret and Dufour on unsteady MHD flow past an infinite vertical porous plate with thermal radiation [9]. Gbadeyan, et al. [10] studied the heat and mass transfer for Soret and Dufour effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field. Soret and Dufour effects on transient MHD flow past a semi-infinite vertical porous plate with chemical reaction was investigated by Shivaiah and Anand [11].

Generally, the thermal-diffusion (Soret) and the diffusion-thermo (Dufour) effects are of smaller order of magnitude than the effects prescribed by Fick's laws and are often neglected in heat and mass transfer processes by many researchers. The effects of Soret for instance has been used for isotope separation. Subhakar and Gangadhar [12] investigated the effects of Soret and Dufour on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source or heat sink. Olanrewaju [13] studied similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation. likewise, Makinde and Mutuku [14] examined the effect of the complex interaction between the electrical conductivity of the conventional base fluid and that of the nanoparticles under the influence of magnetic field in a boundary layer flow with heat transfer over a convectively heated flat surface using numerical approach called Runge–Kutta–Fehlberg method with shooting technique. Prabhakar [15] examined radiation and viscous dissipation effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with hall current in the presence of chemical reaction. The thermal radiation on the flow and heat transfer process is

of great influence in the design of many advanced energy conversion system operating at higher temperature. Thermal radiation within this system is usually as a result of emission by hot walls and the working fluid, Seigel and Howell [16]. Effect of radiation and soret in the presence of heat source/sink on unsteady MHD flow past a semi-infinite vertical plate was studied by Srihari and Srinivas [17]. Makinde, et al. [18] examined combined effects of buoyancy force, convective heating, Brownian motion, thermophoresis and magnetic field on stagnation point flow and heat transfer due to nanofluid flow towards a stretching/shrinking sheet. They observed that both the skin friction coefficient and the local Sherwood number decrease, while the local Nusselt number increases with increasing intensity of buoyancy force and noted that dual solution exists for shrinking case.

Motivated by the above literatures and the numerous possible industrial applications of the MHD problems operating at high temperatures as a result of viscous dissipation, as used in isotope separation, MHD generators, polymer technology, purification of crude oil, fluid droplets sprays and others, it is of paramount interest in this paper to investigate the effects of Thermal Radiation, Soret and Dufour, Viscous dissipation, and other thermo-physical properties on an unsteady heat and mass MHD fluid flow.

In this paper, our concern is to use the numerical Method of lines (MOL) to solve the equations of continuity, linear momentum, energy and diffusion, which govern the flow field. The behaviours of the velocity, temperature and concentration profiles, coefficient of skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

2. MATHEMATICAL MODEL AND ANALYSIS

Consider an unsteady two-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects. The x'-axis is taken along the vertical infinite plate in the upward direction and y'-axis normal to the plate as in Fig. 1 and since the plate is considered infinite in x'-direction, all flow become self-similar away from the leading edge. Therefore, all the physical variable become function of t' and y' only. The effects of soret, dufour and viscous dissipation are considered. By applying Boussinesq's approximation, the flow field is governed by the following boundary layer equations, Alao, et al. [1]:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty)$$
$$+ g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'}$$
$$+ \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{D_m K_T \partial^2 C}{C_s c_p \partial y'^2} \tag{3}$$

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y'^2} -$$
$$K_r'^2(C - C_\infty) + \frac{D_m K_T \partial^2 T}{T_m \partial y'^2} \tag{4}$$

Subject to the conditions:

$$u' = U_0, \ T = T_\omega + \psi(T_\omega - T_\infty)e^{n't'} \tag{5}$$

$$C = C_\omega + \psi(C_\omega - C_\infty)e^{n't'} \text{ at } y' = 0 \tag{6}$$

$$u' \to 0, \ T \to T_\infty, \ C \to C_\infty, \text{ as } y' = \infty, \tag{7}$$

where u' and v' are velocity components in x' and y' directions respectively, t'- the time, C- the dimensional concentration, c_p- the specific heat at constant pressure, D_m- the mass diffusivity, g- the acceleration due to gravity, k_r'- the chemical reaction parameter, α- the fluid thermal diffusivity, β- the thermal expansion coefficient, β*- the concentration expansion coefficient, μ- the coefficient of viscosity, ρ- the fluid density, K_T – the thermal diffusion ratio, T_m- the mean fluid temperature, T_∞ - the free stream dimensional temperature, C_∞ -the free stream dimensional concentration, σ – the electrical conductivity of the fluid, B_0 - the external imposed magnetic field strength in y' direction, q_r – the radiative heat flux, C_s - the concentration susceptivity, v – the viscosity, U_0- the scale of free stream velocity, T_ω- the wall dimensional temperature, C_ω - the wall dimensional concentration, T_∞ - the free stream dimensional temperature, C_∞- the free stream dimensional concentration, n'- the constant.

From the continuity equation (1), it is obvious that suction velocity normal to the plate can either be a constant or function of time. We

consider a case when it is function of both constant and time, hence it is expressed as

$$v' = -V_0 \left(1 + \varepsilon A e^{n't'}\right) \tag{8}$$

where A is a real positive constant, ε and εA are small values less than unity i.e. $\varepsilon \ll 1, \varepsilon A \ll 1$ and V_0 is a non-zero positive constant (the scale of suction velocity at the plate surface), the negative sign indicates that the suction is towards the plate.

In order to simplify the radiative heat flux on the flow, Rosseland diffusion approximation is considered as reported in Adegbie and Fagbade [19] such that;

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y'} \tag{9}$$

where σ_s is the Stefan-Boltzman constant and k_e is the mean absorption coefficient and the fluid is considered to be optically thin.

If the temperature difference within the flow is sufficiently small, then equation (9) can be linearized by expanding T^4 in the Taylor series about T_∞ as follows:

Let $f(t) = T^4$. The Taylor series expansion of $f(t)$ about $T = T_\infty$ is given by

$$f(t) = f(T_\infty) + f'(T_\infty)(T - T_\infty) + f''(T_\infty)\frac{(T-T_\infty)^2}{2!} + \cdots \tag{10}$$

Neglecting higher order terms, equation (10) becomes:

$$f(t) = f(T_\infty) + f'(T_\infty)(T - T_\infty)$$

Hence

$$T^4 = T_\infty^4 + 4T_\infty^3 (T - T_\infty) = 4T_\infty^3 T + 3T_\infty^3 \tag{11}$$

Substituting equation (11) into equation (9) gives:

$$q_r = -\frac{4\sigma_s}{3K_e} \frac{\partial}{\partial y'}(4T_\infty^3 T - 3T_\infty^4) \tag{12}$$

Substituting equation (12) into equation (3) gives:

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y'^2} + \frac{1}{\rho c_p} \frac{4\sigma_s}{3K_e} \frac{\partial^2}{\partial y'^2}$$
$$(4T_\infty^3 T - 3T_\infty^4) + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y}\right)^2 + \frac{D_m K_T \partial^2 C}{C_s c_p \partial y'^2} \tag{13}$$

By algebraic simplification, equation (13) reduces to:

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma_s T_\infty^3}{3\rho c_p K_e} \frac{\partial^2 T}{\partial y'^2} +$$
$$\frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y}\right)^2 + \frac{D_m}{C_s} \frac{K_T}{c_p} \frac{\partial^2 C}{\partial y'^2} \tag{14}$$
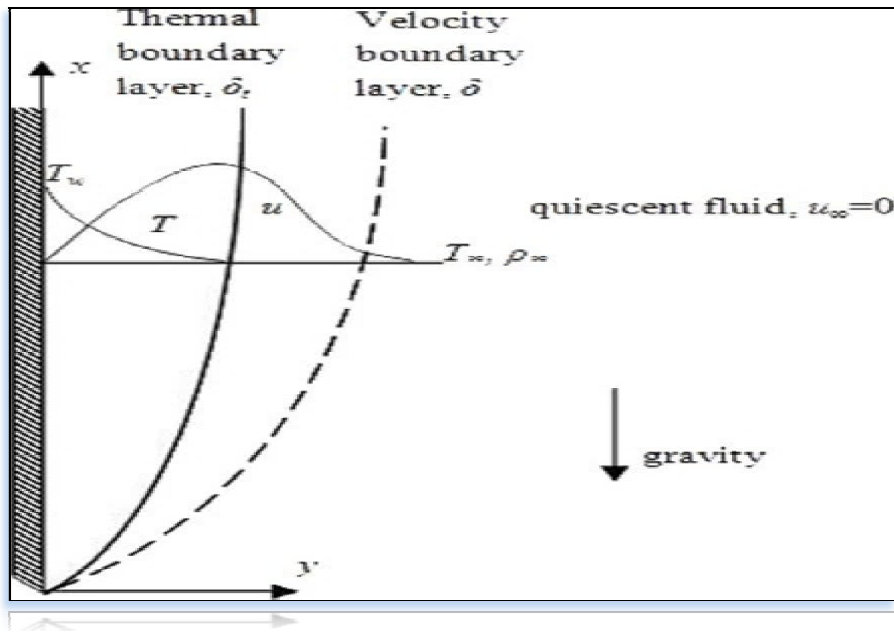

Fig. 1. The configuration of the problem, Alao, et al. [1]

where $\frac{\partial^2}{\partial y^2}(3T_\infty^4) = 0$, as T_∞ is a free stream dimensional temperature which is a constant.

In order to transform the governing equations and the boundary conditions into dimensionless forms, the following non-dimensional quantities are introduced, Alao, et al. [1], Rao, et al. [20], Moorthy and Senthilvadivu [21]

$$u = \frac{u'}{U_0}, \quad y = \frac{V_0 y'}{v}, t = \frac{V_0^2 t'}{v}, n = \frac{vn'}{V_0^2},$$

$$\theta = \frac{T - T_\infty}{T_\omega - T_\infty}, \phi = \frac{C - C_\infty}{C_\omega - C_\infty}, P_r = \frac{v\rho c_p}{k} = \frac{v}{\alpha},$$

$$S_c = \frac{v}{D}, \quad G_r = \frac{g\beta v(T_\omega - T_\infty)}{U_0 V_0^2} \tag{15}$$

$$G_m = \frac{g\beta^* v (C_\omega - C_\infty)}{U_0 V_0^2}, E_c = \frac{U_0^2}{c_p(T_\omega - T_\infty)},$$

$$k_r^2 = \frac{k_r^2 v}{V_0^2}, R = \frac{16\sigma_s T_\infty^3}{3k_e k}, M = \frac{\sigma B_0^2 v}{\rho V_0^2},$$

$$D_u = \frac{D_m K_T(C_\omega - C_\infty)}{C_s c_p v(T_\omega - T_\infty)}, S_r = \frac{D_m K_T(T_\omega - T_\infty)}{T_m v(C_\omega - C_\infty)} \tag{16}$$

In view of equations: (8), (15) and (16), equations: (2), (4) and (14), are algebraically simplified to the following dimensionless forms, Alao, et al. [1]:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} =$$
$$\frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - M^2 u \tag{17}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} =$$
$$\left(\frac{1+R}{Pr}\right)\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + D_u\frac{\partial^2 \phi}{\partial y^2} \tag{18}$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \phi}{\partial y} =$$
$$\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - k_r^2 + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{19}$$

where $G_r, G_m, Pr, R, Ec, Sc, k_r, M, Du$ and Sr are the thermal Grashof number, modified Grashof number, Prandtl number, Radiation parameter, Eckert number, Schmidt number, Chemical reaction parameter, Magnetic parameter, Dufour number and Soret number respectively. u- the velocity profile of the flow, θ- the temperature profile of the flow and ϕ- the concentration profile of the flow, each as a function of y and t.

The transformed boundary conditions are:

$$u(y,t) = 1, \quad \theta(y,t) = 1 + \varepsilon e^{nt},$$
$$\phi(y,t) = 1 + \varepsilon e^{nt} \text{ at } y = 0, \tag{20}$$

$$u(y,t) \to 0, \quad \theta(y,t) \to 0,$$
$$\phi(y,t) \to 0, \quad at \quad y \to \infty \tag{21}$$

3. METHOD OF LINES (MOL)

The basic idea of the MOL is to replace the spatial (boundary value) derivatives in the PDE with algebraic approximations, Biazar and Nomidi [22], Shiesser [23], Knapp [24]. Once this is done, only the initial value variable, typically time in a physical problem, remains. Then, with only one remaining independent variable, we have a system of ODEs that approximates the original PDE. Any suitable integration algorithm for initial value ODEs can now be used to compute an approximate numerical solution to the PDE.

Before applying the method of lines to equations: (17) - (19) subject to the boundary conditions in equations: (20) - (23), we, first of all, adopt the approximations below, to decouple and linearize equations: (17) – (19). Approximating θ, ϕ in equation (17), $\frac{\partial u}{\partial y}, \frac{\partial^2 \phi}{\partial y^2}$ in equation (18), and $\frac{\partial^2 \theta}{\partial y^2}$ in equation (19), all to be unity, Chung [25].

In view of the approximations adopted, equations (17) – (19), reduce to:

$$\frac{\partial u}{\partial t} = (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + G_r + G_m - M^2 u \tag{22}$$

$$\frac{\partial \theta}{\partial t} = (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} + \left(\frac{1+R}{Pr}\right)\frac{\partial^2 \theta}{\partial y^2} + Ec + D_u \tag{23}$$

$$\frac{\partial \phi}{\partial t} = (1 + \varepsilon A e^{nt})\frac{\partial \phi}{\partial y} + \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - k_r^2 + Sr \tag{24}$$

Applying the method of lines to equation (22), we discretize the partial derivative in space variable y, to result in approximating system of ODEs in variable t, thus we have:

$$\left(\frac{du}{dt}\right)_i = (1 + \varepsilon A e^{nt})\frac{(u_{i+1} - u_{i-1})}{2h} +$$
$$\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + G_r + G_m - M^2 u_i \tag{25}$$

Simplifying the right-hand side of equation (25) gives:

$$\left(\frac{du}{dt}\right)_i = \alpha_1 u_{i-1} + \alpha_2 u_i + \alpha_3 u_{i+1} + \alpha_4 \tag{26}$$

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Where

$$\alpha_1 = \left(\frac{1}{h^2} - \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \alpha_2 = M^2 - \frac{2}{h^2}, \alpha_3 = \left(\frac{1}{h^2} + \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \ \alpha_4 = G_r + G_m, i = 1,2,\dots N \tag{27}$$

Equations (26) – (27) can be solved iteratively using the boundary conditions $u(0,t) = 1$ and $u(\infty,t) = 0$ in equations (20) – (21)

For $i = 1, 2, \dots N$, $u(0,t) = u_0(y,t) = 1$ and $u(\infty,t) \approx u(N+1,t) = 0$, equation (26) can be written as;

$$\begin{aligned}
\dot{u}_1 &= \alpha_1 + \alpha_2 u_1 + \alpha_3 u_2 + \alpha_4 \\
\dot{u}_2 &= \alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 + \alpha_4 \\
\dot{u}_3 &= \alpha_1 u_2 + \alpha_2 u_3 + \alpha_3 u_4 + \alpha_4 \\
\dot{u}_4 &= \alpha_1 u_3 + \alpha_2 u_4 + \alpha_3 u_5 + \alpha_4 \\
\dot{u}_N &= \alpha_1 u_{N-1} + \alpha_2 u_N + \alpha_4
\end{aligned} \tag{28}$$

The system in equation (28), in matrix form is given as:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_{N-1} \\ \dot{u}_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{bmatrix} + \begin{bmatrix} \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \vdots \\ \alpha_4 \end{bmatrix} \tag{29}$$

where the coefficients α_1, α_2, α_3 and α_4 are given by equation (27) and $\dot{u}_i = \left(\frac{du}{dt}\right)_i$

In a similar way, equation (23) becomes:

$$\left(\frac{d\theta}{dt}\right)_i = \left(\frac{1+R}{h^2 Pr} - \frac{(1+\epsilon Ae^{nt})}{2h}\right)\theta_{i-1} + -2\left(\frac{1+R}{h^2 Pr}\right)\theta_i + \left(\frac{1+R}{h^2 Pr} + \frac{(1+\epsilon Ae^{nt})}{2h}\right)\theta_{i+1} + (E_c + Du) \tag{30}$$

$$\left(\frac{d\theta}{dt}\right)_i = \beta_1\theta_{i-1} + \beta_2\theta_i + \beta_3\theta_{i+1} + \beta_4 \tag{31}$$

where

$$\beta_1 = \left(\frac{1+R}{h^2 Pr} - \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \beta_2 = -2\left(\frac{1+R}{h^2 Pr}\right), \beta_3 = \left(\frac{1+R}{h^2 Pr} + \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \beta_4 = E_c + Du, i = 1,2,\dots N \tag{32}$$

Equations (31) – (32) can be solved iteratively using the boundary conditions $\theta(0,t) = 1$ and $\theta(\infty,t) = 0$ in equations (20) – (21)

For $i = 1, 2, \dots, N$, $\theta(0,t) = 1 + \varepsilon Ae^{nt}$, $\theta(\infty,t) \approx \theta(N+1,t) = 0$, equation (31) is written in matrix form:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{N-1} \\ \dot{\theta}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 + \varepsilon Ae^{nt} \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N-1} \\ \theta_N \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_4 \\ \beta_4 \\ \beta_4 \\ \vdots \\ \beta_4 \end{bmatrix} \tag{33}$$

where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are given by equation (32) and $\dot{\theta}_i = \left(\frac{d\theta}{dt}\right)_i$

Similarly, equation (24) becomes:

$$\left(\frac{d\phi}{dt}\right)_i = \left(\frac{1}{h^2 Sc} - \frac{(1+\varepsilon Ae^{nt})}{2h}\right)\phi_{i-1} + \frac{-2}{h^2 Sc}\phi_i + \left(\frac{1}{h^2 Sc} + \frac{(1+\varepsilon Ae^{nt})}{2h}\right)\phi_{i+1} + (S_r - K_r^2) \tag{34}$$

$$\left(\frac{d\phi}{dt}\right)_i = \gamma_1\phi_{i-1} + \gamma_2\phi_i + \gamma_3\phi_{i+1} + \gamma_4 \tag{35}$$

where

$$\gamma_1 = \left(\frac{1}{h^2 Sc} - \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \gamma_2 = \frac{-2}{h^2 Sc}, \gamma_3 = \left(\frac{1}{h^2 Sc} + \frac{(1+\varepsilon Ae^{nt})}{2h}\right), \gamma_4 = S_r - K_r^2, i = 1,2,\dots N \tag{36}$$

Equations (35) – (36) can be solved iteratively using the boundary conditions $\phi(0,t) = 1$ and $\phi(\infty,t) = 0$ in equations (20) – (21)

For $i = 1,2,\dots N$, $\phi(0,t) = 1 + \varepsilon Ae^{nt}$, $\phi(\infty,t) \approx \phi(N+1,t) = 0$, equation (35) is written in matrix form:

$$
\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \vdots \\ \dot{\phi}_{N-1} \\ \dot{\phi}_N \end{bmatrix} =
\begin{bmatrix}
\gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & \cdots & 0 & 0 \\
0 & \gamma_1 & \gamma_2 & \gamma_3 & 0 & \cdots & 0 & 0 \\
0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & \gamma_1 & \gamma_2
\end{bmatrix}
\begin{bmatrix} 1+\varepsilon Ae^{nt} \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \\ \phi_N \end{bmatrix} +
\begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \end{bmatrix} \tag{37}
$$

where the coefficients $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are given by equation (36) and $\dot{\phi}_i = \left(\frac{d\phi}{dt}\right)_i$

In practical engineering application, the physical quantities of practical values are the local skin friction due to velocity profiles, rate of heat transfer (Nusselt number) due to temperature profiles and rate of mass transfer (Sherwood number) due to concentration profiles.

The three physical quantities at the plate are given in as, Alao, et al. [1]:

Skin friction coefficient:

$$c_f = \frac{\tau'_w}{\rho u_0 v_0} = \left(\frac{\partial u}{\partial y}\right)\Big|_{y=0} \tag{38}$$

Nusselt Number: $Nu = -\frac{x\left(\frac{\partial T}{\partial y}\right)}{(T_w - T_\infty)}\Big|_{y'=0} \tag{39}$

Sherwood Number: $Sh = -\frac{x\left(\frac{\partial C}{\partial y}\right)}{(C_w - C_\infty)}\Big|_{y'=0} \tag{40}$

Using non-dimensional quantities defined in equations (15) – (16), the Nusselt Number and Sherwood Number are simplified as:

$$Nu/Re_x = -\left(\frac{\partial\theta}{\partial y}\right)\Big|_{y=0} \tag{41}$$

And

$$\frac{Sh}{Re_x} = -\left(\frac{\partial\phi}{\partial y}\right)\Big|_{y=0} \tag{42}$$

where $Re_x = \frac{V_0 x}{v}$ is the local Reynolds number.

4. RESULTS AND DISCUSSION

In this paper, a numerical approach called method of lines (MOL) has been used to solve the transformed equations (22) – (24) subject to the boundary conditions (20) and (21). With this approach, the effects of governing flow parameters on the velocity profile, temperature profile and concentration profile are discussed and presented by the graphs. Also, the computational values for coefficient of skin friction, Nusselt number and Sherwood number for different values of flow parameters are discussed and presented in tables. For the analysis of the results, various values of flow controlling parameters: $G_r = G_m = 2.0, P_r = 0.71, E_c = 0.001, S_C = 0.6, R = k_r = S_r = 0.5, n = 0.1, t = 1.0, h = 0.1, \varepsilon = 0.02$, are used for computations. Thus, all the graphs and tables correspond to these values unless otherwise stated. The system of ODEs obtained from equations (29), (33) and (37) are solved uing with MATLAB CODE (ode45 and ode15s) depending on the stiffness of the equations, with iteration $i = 1,2,3$.

In the absence of Soret (S_r) and Dufour (D_u) parameters, the computational results obtained in this paper agrees with the results of Rao, et al [17] and Alao, et al [16], as shown in Table 1.

Table 1. Computational values for local skin-friction, $u'(0)$, and local Nusselt number, $-\theta'(0)$, for different values of thermal radiation parameter (R) when $M = 0.5, S_r = Du = 0, E_c = 0.001, G_m = G_r = 2.0, P_r = 0.71, A = K_r = 0.5, Sc = 0.6, n = 0.1, t = 1.0, h = 0.1, \varepsilon = 0.02$

R	Present Study		Alao, et al.		Rao, et al.	
	$u'(0)$	$-\theta'(0)$	$u'(0)$	$-\theta'(0)$	$u'(0)$	$-\theta'(0)$
0.0	2.1578	0.8337	2.1693	0.8291	2.1664	0.8365
0.5	2.4406	0.6157	2.4657	0.6154	2.4548	0.6139
1.0	2.6325	0.5164	2.6546	0.5087	2.6536	0.5032
2.0	2.9406	0.4183	2.9039	0.4017	2.9037	0.4010

Table 2. Computed values of the local skin-friction for various values of Soret number (S_r) and Dufour number (D_u) when $M = 0.1, h = 0.1, P_r = 0.71, G_r = G_m = 2.0, R = 0.5, t = 1.0, n = 0.1, \varepsilon = 0.02 \ k_r = 0.5$ and the computed values of the local nusselt number for various values of Soret number (S_r) and Dufour number (D_u) when $\varepsilon = 0.02, A = 0.5, n = 0.1, t = 1.0, h = 0.1, P_r = 0.71, R = 0.5, E_c = 0.001$

		local skin-friction				local nusselt number	
S_r	D_u	Present study	Alao, et al.	S_r	D_u	Present study	Alao, et al.
		$u'(0)$	$u'(0)$			$-\theta'(0)$	$-\theta'(0)$
0.	0	1.7795	1.7539	0.0	0	0.6683	0.6936
0	0.5	1.9090	1.8830		0.5	0.5411	0.5559
	1.0	2.0227	2.0121		1.0	0.4172	0.4183
0.	0	1.9090	1.8907	0.5	0	0.6683	0.6936
5	0.5	2.0227	2.0198		0.5	0.5466	0.5559
	1.0	2.1388	2.1489		1.0	0.5401	0.4183
1.	0	2.0227	2.0274	1.0	0	0.6683	0.6936
0	0.5	2.1388	2.1566		0.5	0.5411	0.5559
	1.0	2.2242	2.2857		1.0	0.4172	0.4183

Table 3. Computed values of the local Sherwood number for various values of soret number (S_r) and dufour number (D_u) when $\varepsilon = 0.02, A = 0.5, n = 0.1, t = 1.0, h = 0.1, Sc = 0.6, k_r = 0.5$

S_r	Du	Present study	Alao et.al	S_r	D_u	Present study	Alao et.al
		$-\phi'(0)$	$-\phi'(0)$			$-\phi'(0)$	$-\phi'(0)$
0.0	0.0	0.6683	0.6936	0.5	0.0	0.8639	0.8634
	0.5	0.5411	0.5559		0.5	0.8639	0.8634
	1.0	0.4172	0.4183		1.0	0.8639	0.8634
0.5	0.0	0.6683	0.6936	0.5	0.0	0.7238	0.6994
	0.5	0.5466	0.5559		0.5	0.6888	0.6994
	1.0	0.5401	0.4183		1.0	0.6482	0.6994
1.0	0.0	0.6683	0.6936	1.0	0.0	0.5445	0.5354
	0.5	0.5411	0.5559		0.5	0.5415	0.5354
	1.0	0.4172	0.4183		1.0	0.5334	0.5354

Table 4. Computed values of the local skin friction and the local Nusselt number for various values of the thermal radiation parameter R when $P_r = 0.71, G_r = G_m = 2.0, S_r = Du = 0.5, t = 1.0, n = 0.1, M = 0.1$

	local skin friction			Nusselt number	
R	Present study	Alao et.al	R	Present study	Alao et.al
	$u'(0)$	$u'(0)$		$-\theta'(0)$	$-\theta'(0)$
0.0	1.7546	1.7940	0.0	0.7330	0.7455
0.4	1.9735	1.9166	0.4	0.6652	0.6533
0.8	2.0032	2.0111	0.8	0.6192	0.6051
1.0	2.0504	2.0512	1.0	0.5814	0.5892

Table 5. Computed value of the local Sherwood number for various values of the thermal radiation parameter (R) when $P_r = 0.71, G_r = G_m = 2.0, S_r = Du = 0.5, t = 1.0, n = 0.1, M = 0.1$

R	Present study	Alao et.al
	$-\phi'(0)$	$-\phi'(0)$
0.0	0.6834	0.6994
0.4	0.6807	0.6994
0.8	0.6647	0.6994
1.0	0.6277	0.6994

5. CONCLUSION

The study has examined the Method of Lines analysis (MOL) on the problem of an unsteady heat and mass transfer flow of an MHD fluid past a semi-infinite vertical plate with viscous dissipation under the influence of thermal radiation, Soret, Dufour and other pertinent flow parameters. The following conclusions are drawn from the study:

(1) As soret number (S_r) and dufour number (D_u) increase, the skin-friction coefficient (C_f) increases while the Nusselt number (Nu) decreases.

(2) As soret number (S_r) increases, the Sherwood number (Sh) reduces drastically while increase in dufour number (D_u) has negligible or no influence on it.

(3) As the thermal radiation parameter (R) increases with an optimized value of magnetic parameter (M), the skin-friction coefficient (C_f) increases while the Nusselt number (Nu) decreases and there is negligible or no influence on the Sherwood number (Sh).

(4) As the thermal radiation parameter (R) increases, both the velocity $u(y,t)$ profile and the temperature profile $\theta(y,t)$ increase. Thus, it is obvious that the thermal radiation intensifies the convective flow.

(5) As soret number (S_r) increases, both the velocity $u(y,t)$ profile and the concentration profile $\phi(y,t)$ increase, while there is negligible or no effect on the temperature distribution.

(6) As dufour number (D_u) increases, both the velocity $u(y,t)$ profile and the temperature profile $\theta(y,t)$ increase.

(7) An increase in the magnetic parameter (M) decreases the velocity profile $u(y,t)$ as a result of a resistive Lorentz force produced by the magnetic field.

(8) As Schmidt number (S_C) and Chemical reaction (K_r) increase, the velocity profile $u(y,t)$ as well as the concentration profile $\phi(y,t)$ decrease.

(9) As Eckert number (E_c) increases, both the velocity $u(y,t)$ profile and the temperature profile $\theta(y,t)$ increase.

(10) As the thermal Grashof number (G_r) and the modified Grashof number (G_m) increase, the velocity profile $u(y,t)$ increases.

(11) It has been observed that, in designing a system where high temperature is needed, such as glass production, propulsion system, plasma physics etc., the effects of thermal radiation parameter, Soret, Dufour, Eckert number, Grashof number, Chemical reaction parameter, Schmidt number and Magnetic parameter should be considered carefully, for optimal performance of such system.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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