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Modeling Volatility of Asset and Volume of Trade Returns in the Nigerian Stock Market in the Presence of Random Level Shifts

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study investigated the impact of volatility shock persistence on the conditional variance in the Nigerian stock returns using symmetric and asymmetric higher order GARCH family models in the presence of random level shifts and non-Gaussian errors. The study utilised Bai and Perron methodology to detect structural breakpoints in the conditional variance of daily stock and volume of trade returns in the Nigerian stock market from 2nd January, 1998 to 22nd March, 2017. The study employed symmetric GARCH (3,2) and GARCH (2,1)-M models to estimate volatility of asset returns, symmetric GARCH (2,2) and GARCH (2,1)-M to model volatility of volume of trade returns and asymmetric EGARCH (2,2), TGARCH (3,2) and PGARCH (2,3) models to measure the volatility of asset returns as well as asymmetric EGARCH (2,1), TGARCH (1,3) and PGARCH (3,2) models to estimate volatility of volume of trade returns. These models were optimally selected using information criteria and log likelihood as the best fitting symmetric and asymmetric GARCH models to estimate the conditional volatility of asset and volume of trade returns in the Nigerian stock market with and without structural breaks. Results revealed that when random level shifts were ignored in volatility models, the shocks persistence were very high

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with long memory and variance explosion. But when the random level shifts were incorporated into the GARCH models, there was a significant reduction in the volatility shocks persistence and long memory. Moreover, volatility half-lives also declined drastically while accounting for these sudden level shifts in variance. The study found asymmetry without leverage effects as well as a positive risk-return tradeoff for both asset and volume of trade returns in the Nigerian stock market. The Nigeria banking reform of 2004, the Global Financial and Economic Crises, as well as other local events in Nigeria, were found to have negative and significant impacts on the Nigerian stock market. The study provided some policy recommendations.

Keywords: Asymmetry; leverage effects; random level shifts; risk-return tradeoff; shock persistence; volatility; Nigeria.

1 Introduction

Volatility modelling and forecasting of stock market returns has become a fertile area of research in statistics, economics and finance over the last two decades. This area has been receiving considerable attention from academics, financial analysts and practitioners because volatility is an important tool for many economic and financial applications including options trading, asset pricing, financial risk management, portfolio selection and optimisation as well as strategic pair trading. Modelling the variance of the errors also improves the efficiency in parameter estimation and the accuracy in interval forecast. The fact that volatility is not directly observable makes many financial analysts to have a keen interest in obtaining accurate estimates of the conditional variance so as to improve risk management, portfolio selection, options pricing and valuation of financial derivatives [1]. However, the accuracy of these estimates cannot be guaranteed when structural breaks in the conditional variance of stock returns are ignored.

Many time series models such as Autoregressive (AR) model, Moving Average (MA) model, Autoregressive Moving Average (ARMA) model and the Autoregressive Integrated Moving Average (ARIMA) model proposed by Box and Jenkins [2] are inadequate in capturing accurately the time-varying and long memory in volatility due to their short memory features. The symmetric Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle [3] and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model extended by Bollerslev [4] then become the most widely used models in studying the volatility of financial return series. Certain stylised facts of stock return such as fat/heavy tails, volatility clustering and volatility persistence can easily be captured by the symmetric GARCH models. However, asymmetry and leverage effects in the return series cannot be captured by the symmetric GARCH models. This led to the extension of other GARCH variants such as asymmetric Exponential GARCH by Nelson [5], Threshold GARCH by Zakoian [6] & Glosten [7] and Power GARCH model by Ding et al. [8] among others. In recent times, several empirical evidences in the financial literature found support for the GARCH-type models.

The crash of the Nigerian stock market as a result of the Global Financial Crisis, economic crisis and other local events have created some level shifts (structural breaks) in the variance of stock return series. Therefore, the conventional GARCH variants which ignore these shifts may not be adequate in obtaining accurate volatility estimates in the Nigerian stock market [9]. This study intends to employ both symmetric and asymmetric higher order GARCH family models with exogenous breaks and heavy-tailed distributions to investigate the impact of volatility shock persistence on the conditional variance due to this crash, the banking reform of 2004 and other internal events on the Nigerian stock market using both daily closing all share index (ASI) and volume of trade (VOT) returns of the Nigerian Stock Exchange (NSE).

The main objective of this study is to investigate the behaviour of stock return volatility in Nigerian stock market in the presence of random level shifts using higher order GARCH family models. This involves examining the NSE stock return series for evidence of volatility clustering, shock persistence, fat-tails distribution, and asymmetry and leverage effects as they provide essential information about the riskiness of

asset and volume of trade returns in the Nigerian stock market. The study also investigates the impact of exogenous breaks on the conditional variance as well as risk-return tradeoff in Nigerian stock returns.
The rest of the paper is organised as follows: Section 2 reviews relevant literature on the subject matter, Section 3 presents data, materials and methods; Section 4 presents and discusses results of empirical findings while section 5 presents conclusion and recommendations.

2 Literature Review

In antiquity, it was generally believed that long memory in volatility was a common phenomenon associated with all financial time series data. Nowadays, researchers argue that it could be the presence of structural breaks (random level shifts) that bring about long memory in a series. Thus, the estimates and conclusions on financial returns and their modelling as long memory would be biased. In studies conducted by Perron [10,11] it was shown that when stationary series are contaminated with structural breaks, the sums of the autoregressive coefficients are biased to unit. In the context of GARCH models, [8] estimate the ARCH model, taking into account the squared returns and absolute returns, and show the existence of long memory. Then the authors proposed the asymmetric power GARCH model (APGARCH), allowing the long memory parameter in the volatility and the asymmetry parameter. Teverovsky and Taqqu [12] presented a method for distinguishing between the effects of level shifts and the effects of long memory. Gourieroux and Jasiak [13] evaluate the relationship between the presence of infrequent breaks and long memory based on the correlogram estimation instead of estimating the fractional parameter. The authors found that non-linear time series with infrequent breaks could have long memory. Therefore, these series and not the fractionally integrated processes with *i.i.d.* innovations would cause the hyperbolic decay of the autocorrelogram. McMillan and Thupayagale [14] found that the estimates of long memory in volatility were sensitive to structural breaks.

There has been a large volume of literature on modelling and forecasting stock market volatility in the presence of exogenous breaks from both developed and emerging economies around the globe. For examples, [15] conducted a study on volatility modelling and concluded that ignoring structural breaks in volatility models increases shock persistence whereas incorporating the structural breaks in the conditional variance of returns reduces the persistence of shocks in volatility models. Malik et al. [16] investigated the volatility shock persistence on the Canadian stock market using GARCH family models. Results showed a reduction in volatility shock persistence when structural breaks were incorporated in the conditional variance while estimating volatility. Hammoudeh & Li [17] also found a significant reduction in shock persistence when structural breaks were incorporated in the conditional variance while predicting volatility in Gulf Arab countries stock markets. Alfreedi et al. [18] investigated volatilities in the Gulf Cooperation Council (GCC) stock markets using asymmetric EGARCH, ICSS-EGARCH, GJR-GARCH, and ICSS-GJR-GARCH models on weekly data for the period 2003-2010. The ICSS-EGARCH and ICSS-GJR-GARCH models took into account the discrete regime shifts in stochastic errors. The finding supported the widely accepted view that accounting for the regime shifts detected by the iterated cumulative sums of squares (ICSS) algorithm in the variance equations reduced the overestimation of volatility shock persistence. The sudden changes were generally associated with global, regional, and domestic, economic as well as political events.

Muhammad & Shuguang [19] investigated the impact of exogenous breaks on the conditional volatility and variance persistence of GARCH family models while employing Bai and Perron multiple breakpoints test procedure to detect structural breakpoints in conditional variance of daily stock returns of seven emerging markets from 1977 to 2014. They employed asymmetric EGARCH (1,1) and TGARCH (1,1) models with and without breaks and found that shock persistence in the conditional variance significantly reduced when level shifts were considered in the conditional volatility of these models. The half-lives to shocks were also found to decline significantly in the presence of these sudden break points.

In Nigeria, published works on volatility modeling of stock returns with exogenous breaks are also well documented in the literature. For examples, [20] examined the monthly exchange rate volatility of naira against US Dollar, British Pounds and European Euro using GARCH family models in the presence of structural breaks. Result showed high persistence of shocks in all the models when sudden breaks were ignored, although the incorporation of structural breaks in the models improved the volatility estimates by reducing shock persistence in most of the estimated models. Dikko et al. [21] conducted a study to model abrupt shift in time series using indicator variable. Seven symmetric and five asymmetric models were considered by incorporating an indicator variable in the variance equation to monitor the changes of some selected Nigerian insurance stocks. The results showed that the daily returns were stationary but not normally distributed and eight out of ten stocks considered for the study showed evidence of ARCH effect. The performance of the different models was evaluated using the RMSE, MAE and MAPE. The model ARCH (1) proved to be the most suitable among the twelve competing volatility models considered. When the regime changes were incorporated into the model, it was found that the highly persistent volatility of the insurance stock return rate was reduced for most of the stocks. Adewale et al. [22] investigated the volatility and asymmetry in the Nigerian stock returns using monthly data for the period from January 1985 to December 2014 while incorporating structural breaks. The data was segmented into pre-structural break period and after break period having identified breakpoints in the series. Results showed higher shock persistence during pre-break sub-period than the post-break sub-period. There was no evidence of leverage effect in the asymmetric GARCH model with or without incorporating the breakpoints in Nigerian stock market.

Recently, [23] investigated the performances of different GARCH models while estimating the volatility of headline and core CPI inflation in Nigeria for the period 1995M01 to 2016M10 using ADF breakpoint testing procedure. They applied both symmetric and asymmetric GARCH variants and observed empirical evidence of shock persistence in both CPI stock returns with the presence of leverages only in the headline CPI return series. The authors concluded that ignoring the role of structural breaks in estimating the volatility of inflation rate in Nigeria will amount to misleading policy prescriptions. Kuhe & Chiawa [9] conducted a study to examine the impact of structural breaks on the conditional volatility of the daily stock returns of eight commercial banks in Nigeria from 17th February, 2003 to 31st September, 2016. The study used symmetric GARCH (1,1), asymmetric EGARCH (1,1) and TGARCH (1,1) models with and without level shifts to estimate volatility. Results showed high volatility shock persistence in the banking stocks when the level shifts were ignored, but when the random level shifts were included in the models, the shock persistence drastically reduced and the volatility half-lives also reduced in the presence of these level shifts. Kuhe [24] found similar results. On modelling volatility in Nigerian stock market using GARCH family models with breaks see also the recent works of [25,26,27,28,29] among others for similar contributions.

From the reviewed literature, it is important to know that researchers who investigated volatility shock persistence in the conditional variance in Nigerian stock market in the presence of structural breaks used either daily or monthly all share index (ASI) from the Nigerian stock exchange and considered only the 2007-2009 global financial crisis on lower order GARCH models. However, it is not only the well-known global financial crisis which started from 2007 to 2009 that affected the Nigerian stock market. The banking reform of 2004 in Nigeria and other internal factors can also affect the stock market [19]. This study therefore, extends the existing literature by employing both symmetric and asymmetric higher order GARCH family models with exogenous breaks and heavy-tailed distributions to investigate the impact of volatility shock persistence on the conditional variance while accounting for the global financial crisis, the banking reform of 2004 and other internal events on the Nigerian stock market using more recent data on both daily closing all share index (ASI) and volume of trade (VOT) returns of the Nigerian Stock Exchange (NSE). This study uses methodology slightly different from the ones mentioned in the literature as it employs Bai & Perron multiple breaks testing procedure that detects breakpoints in the entire data set of the Nigerian stock returns. Once the breakpoints are detected, they are accounted for by creating an indicator (dummy) variable which takes the value zero for stable and one for unstable economy; these dummies are incorporated in the conditional variance equations of all the symmetric and asymmetric GARCH models to avoid overestimation of volatility shock in the conditional variance.

3. Materials and Methods

3.1 Data and data transformation

The data used in this research work are the daily closing all share index (ASI) and volume of trade (VOT) of the Nigerian Stock Exchange (NSE) obtained from www.nse.ng.org for the period 2nd January, 1998 to 22nd March, 2017 making a total of 4725 observations each. The daily returns r_t are calculated as:

$$r_t = \ln \Delta P_t \times 100 \tag{3.1}$$

where r_t denotes the stock return series, Δ is the first difference operator and P_t denotes the closing market index at the current day (t).

3.2 Descriptive and normality test statistics

The sample standard deviation ($\hat{\sigma}$) of returns over a given period of time is computed as:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})} \tag{3.2}$$

where $\bar{r}$ is the sample mean return defined by:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i \tag{3.3}$$

Jarque and Bera [30,31] proposed a normality test which is goodness-of fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test is usually used to test the null hypothesis that the series is normally distributed. Given a return series $\{r_t\}$ the test statistic JB is defined as:

$$JB = \frac{T}{6}\left(g_1^2 + \frac{1}{4}(g_2 - 3)^2\right) \tag{3.4}$$

where g_1 is the sample skewness given as:

$$g_1 = \frac{\mu_3}{\mu_2^{3/2}} = T^{1/2}\sum_{t=1}^{T}(r_t - \bar{r})^3 \Bigg/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{3/2} \tag{3.5}$$

and g_2 is the sample kurtosis defined by:

$$g_2 = \frac{\mu_4}{\mu_2^2} = T\sum_{t=1}^{T}(r_t - \bar{r})^4 \Bigg/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{2} \tag{3.6}$$

where T is the number of observations and $\bar{r}$ is the sample mean. The normal distribution has a skewness equal to 0 with a kurtosis of 3. The JB normality test checks the following pair of hypothesis:

H_0: $\hat{\mu}_3 = 0$ and $\hat{\mu}_4 = 0$ (i.e., r_t is normally distributed) against the alternative H_1: $\hat{\mu}_3 \neq 0$ and $\hat{\mu}_4 \neq 0$ (i.e., r_t is not normally distributed). The test rejects the null hypothesis if the p-value of the JB test statistic is less than $\alpha = 0.05$ level of significance.

3.3 Dickey-fuller generalized least squares (DF GLS) unit root test

Elliot et al. [32] optimise the power of the Augmented Dickey-Fuller (ADF) unit root test by de-trending, if y_t is the series under investigation, then, the DF GLS test is based on testing the following hypothesis:

$H_0: \psi = 0$ (the series contains unit root) against
$H_1: \psi < 0$ (the series is stationary) in the following regression

$$\Delta y_t^d = \psi_0 y_{t-1}^d + \psi_1 \Delta y_{t-1}^d + \cdots + \psi_{p-1} \Delta y_{t-p+1}^d + u_t \quad (3.7)$$

where y_t^d is the detrended series. Detrending depends on whether a constant or a constant and trend are included in the model. Taking the more general case.

$$y_t^d = y_t - \hat{\beta}_0 - \hat{\beta}_1 t \quad (3.8)$$

where $(\hat{\beta}_0, \hat{\beta}_1)$ are obtained by regressing $\bar{y}$ on a constant and time trend (the latter deterministic variable denoted as $\bar{z}$), where

$$\left.\begin{array}{l} \bar{y} = [y_1, (1-\bar{\alpha}L)y_2, \ldots, (1-\bar{\alpha}L)y_T] \\ \bar{z} = [z_1, (1-\bar{\alpha}L)z_2, \ldots, (1-\bar{\alpha}L)z_T] \end{array}\right\} \quad (3.9)$$

and $z_t = (1, t)'$, $\quad \bar{\alpha} = 1 + \bar{c}/T$

where T represents the number of observations for y_t and $\bar{c}$ be fixed at -7 in the model with only a constant (drift) and at -13.7 when both a constant and trend term enter the ADF regression. Elliot et al. [32] show that de-trending in this way produces a test that has good power properties.

3.4 Heteroskedasticity test

To test for heteroskedasticity or ARCH effect in the residuals of returns, we apply the Lagrange Multiplier (LM) test due to Engle [3]. The procedure of performing the Engle's LM test is to first obtain the residuals e_t from an ordinary least squares regression of the conditional mean equation which could be an AR, MA or ARMA model that best fit the data. For instance, in an ARMA (1,1) model, the conditional mean equation is specified as:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \quad (3.10)$$

where r_t is the return series, $\phi_1$and θ_1 are the coefficients of the AR and MA terms while ε_t is the random error term. Having obtained the residuals e_t, we then regress the squared residuals on a constant and q lags such as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \alpha_3 e_{t-3}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t \quad (3.11)$$

The null hypothesis of no ARCH effect up to lag q is then formulated as follows:

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_q$ versus the alternative $H_1: \alpha_i > 0$ for at least one $i = 1,2,3, \ldots, q$.

There are two test statistics for the joint significance of the q-lagged squared residuals. The F-statistic and the number of observations times R-squared (nR^2) from the regression. The F-statistic is estimated as:

$$F = \frac{SSR_0 - SSR_1/q}{SSR_1(n-2q-1)} \quad (3.12)$$

$$\text{where } SSR_1 = \sum_{t=q+1}^{T} e_t^2, SSR_0 = \sum_{t=q+1}^{T} (r_t^2 - \bar{r})^2 \text{ and } \bar{r} = \frac{1}{\mathrm{n}} \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{r}_{\mathrm{t}}^2 \quad (3.13)$$

$\hat{e}_t$ is the residual obtained from least squares linear regression, $\bar{r}$ is the sample mean of r_t^2. The nR^2 is evaluated against $\chi^2(q)$ distribution with q degrees of freedom under H_0. The decision is to reject the null hypothesis of no ARCH effect in the residuals of returns if the p-values of the F-statistic and nR^2 statistic are less than $\alpha = 0.05$.

3.5 Bai and Perron multiple breakpoints test

Bai and Perron [33] developed a multiple structural breakpoints testing procedure which predicts persistently several shifts in variance. The power of the test was strengthened by Bai and Perron [34] which made the test more efficient. The model considered is the multiple linear regression model with m breaks or $m + 1$ regimes.

$$y = x_i^T \beta_i + u_t \tag{3.14}$$

$$y_i = x_i^T \beta_i + z_i^T \delta + u_t \tag{3.15}$$

where $u_i \sim iid(0, \sigma^2)$, $i = 1, 2, 3, \dots, n$ and y_i is the response variable at time i and $x_i = [1, x_{i2}, x_{i3}, \dots, x_{ik}]^T$ is a vector of order $k \times 1$ of independent variables having one as its initial value and β_i is also $k \times 1$ vector of coefficients. The hypothesis for random level shift is:

H_0: $\beta_i = \beta_0$ for $i = 1, 2, 3, \dots, n$ (i.e., there is no structural break in the series) versus alternative that with the random level shift in time the vector of coefficients also changes, also assuming that they have no stochastic behaviour as a departure from the null hypothesis. i.e.,

$$\|x_i\| = \boldsymbol{O}(1) \text{ and that } \frac{1}{n}\sum_{i=1}^{n} x_i x_i^T \to Z$$

where Z represents a finite matrix. This expression permits the detection of multiple breakpoints in data and once the breakpoints are recognised, they will be incorporated into each GARCH model in order to avoid spurious results. This same procedure is implemented in this study to detect multiple break points in the given stock return series before moving forwards.

3.6 Model specification

The following heteroskedastic models are specified for this study.

3.6.1 The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Bollerslev [4] extended the ARCH model of Engle [3] to Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. A GARCH (p, q) process is specified as:

$$r_t = \mu_t + \varepsilon_t \tag{3.16}$$

$$\varepsilon_t = \sigma_t e_t; \quad e_t \sim N(0, 1) \tag{3.17}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.18}$$

In order to account for date wise structural breaks in the conditional variance of returns, we incorporate a dummy variable in the conditional variance of GARCH (p,q) model. The GARCH (p,q) model with dummy variable in the conditional variance is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{l=1}^{n_c} \phi_c DUM_{c,j,t} \tag{3.19}$$

where ε_t is the innovation/shock at day t and it follows heteroskedastic error process, σ_t^2 is the volatility at day t (conditional variance), ε_{t-i}^2 is squared innovation at day $t-i$, ω is a constant term, μ_t can be any adapted model for the conditional mean; p is the order of the autoregressive GARCH term; q is the order of the moving average ARCH term, n_c denotes the total numbers of date wise changes in market c, DUM is the dummy variables added to the conditional variance which takes value 1 as the sudden shift comes out in conditional volatility and elsewhere it takes value zero. A GARCH (p,q) process is stationary if and only if

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \tag{3.20}$$

3.6.2 GARCH-in-Mean (GARCH-M) Model

The GARCH-in-mean (GARCH-M) model was proposed by Engle et al. [35] to makes a significant change to the role of time-varying volatility by explicitly relating the level of volatility to the expected return. The mean equation of a simple GARCH-in-mean model is specified as:

$$r_t = \mu + \lambda\sigma_t^2 + \varepsilon_t; \quad \varepsilon_t = \sigma_t e_t, \qquad e_t \sim N(0,1) \tag{3.21}$$

where μ and λ are constants. The parameter λ is called the risk premium parameter. A positive λ indicates that the return is positively related to its past volatility.The variance equation of a simple GARCH (2,1)-in-mean model is specified as:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \beta_1\sigma_{t-1}^2 \tag{3.22}$$

The GARCH-M model is used in financial applications where the expected return on an asset is related to the expected asset risk. The estimated coefficient on the expected risk is a measure of the risk-return tradeoff.

The GARCH (2,1)-M model which incorporates structural breaks in the conditional variance is given by:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \beta_1\sigma_{t-1}^2 + \sum_{l=1}^{n_c} \phi_c DUM_{c,j,t} \tag{3.23}$$

The accompanying variables are as earlier defined.

3.6.3 Asymmetric GARCH Models

There are several GARCH variants proposed in the literature. However, in this work we consider only three asymmetric GARCH models: exponential GARCH, threshold GARCH and power GARCH models.

3.6.3.1 The Exponential GARCH (EGARCH) Model

The EGARCH model was the first asymmetric GARCH model proposed by Nelson [5] to overcome some weaknesses of the basic GARCH model in handling financial time series, particularly to allow for asymmetric effects between positive and negative asset returns. The EGARCH (p, q) model is expressed as:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right\} + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}} \right] \tag{3.24}$$

where γ represents the asymmetric coefficient in the model, β coefficient represents the measure of persistence, mostly less than one but as its value approaches unity the persistence of shock increases. To facilitate the sudden shifts in variance we introduce a dummy variable in the specification of the conditional variance EGARCH (p,q) model as follows:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left\{\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right\} + \sum_{j=1}^{q} \beta_j \ln\left(\sigma_{t-j}^2\right) + \sum_{k=1}^{r} \gamma_k \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] + \sum_{l=1}^{n_c} \phi_{c,l} DUM_{c,l,t} \tag{3.25}$$

3.6.3.2 The threshold GARCH (TGARCH) model

The Threshold ARCH (TARCH) and Threshold GARCH (TGARCH) were introduced independently by Zakoian [6] & Glosten [7]. The generalised specification of TGARCH (p,q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{k=1}^{v} \gamma_k \varepsilon_{t-k}^2 S_{t-k}^- \tag{3.26}$$

where $S_t^- = 1$ if $\varepsilon_t < 0$ and 0 otherwise. In this model, good news, $\varepsilon_{t-i} > 0$, and bad news, $\varepsilon_{t-i} < 0$, have different effects on the conditional variance; good news has impact on α_i, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and we say that there is a leverage effect for the $i - th$ order. If $\gamma \neq 0$, the news impact is asymmetric.

The TGARCH (p,q) model with dummy variable for structural break points is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{k=1}^{v} \gamma_k \varepsilon_{t-k}^2 S_{t-k}^- + \sum_{l=1}^{n_c} \phi_c DUM_{c,j,l} \tag{3.27}$$

3.6.3.3 The power GARCH (PGARCH) model

The Power ARCH (PARCH) model was first introduced by Taylor [36] & Schwert [37] where the standard deviation is modeled rather than the variance. This model along with several other models is generalised in [8] with the power GARCH specification. In PGARCH model, the power parameter δ of the standard deviation can be estimated rather than imposed, and the optional γ parameters are added to capture asymmetry of up to order r. The PGARCH (p,q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^\delta \tag{3.28}$$

We incorporate the sudden shifts in variance into the PGARCH model of [8] to include a dummy variable in the conditional variance as given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^\delta + \sum_{l=1}^{n_c} \phi_c DUM_{c,j,l} \tag{3.29}$$

3.7 Model order selection using information criteria

GARCH model order selection involves selecting a model order that minimises one or more information criteria evaluated over a range of model orders. In this work, we employed Akaike Information Criterion (AIC) due to Akaike [38], Schwarz information Criterion (SIC) due to Schwarz [39], and Hannan-Quinn Information Criterion (HQC) due to Hannan [40]. The information criteria are given below:

$$AIC(P) = -2\ln(L) + 2P \tag{3.30}$$

$$SIC(P) = -2\ln(L) + Pln(T) \tag{3.31}$$

$$HQC(P) = 2\ln[\ln(T)]\,P - 2\ln\,(L) \tag{3.32}$$

where P is the number of free parameters to be estimated in the model, T is the number of observations and L is the maximum likelihood function for the estimated model defined by:

$$\left.\begin{aligned} L &= \prod_{i=0}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2} exp\left[-\sum_{i=1}^{n}\frac{(y_i - f(x))^2}{2\sigma_i^2}\right] \\ \ln\,(L) &= In\left[\prod_{i=1}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - f(x))^2}{\sigma_i^2} \end{aligned}\right\} \tag{3.33}$$

Thus given a set of estimated GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and highest log likelihood value.

3.8 Error distributions and GARCH models estimation

Given any error distribution, GARCH models are typically estimated by the method of maximum likelihood. The five error distributions employed in this work are given as:

(i) Normal (Gaussian) distribution (ND) is given as:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}, -\infty < z < \infty \tag{3.34}$$

The normal distribution to the log likelihood for observation t is given as:

$$l_t = \frac{-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\sigma_t^2 - \frac{1}{2}(y_t - X_t'\theta)^2}{\sigma_t^2} \tag{3.35}$$

(ii) Student-t Distribution (STD) is given as:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \tag{3.36}$$

For student's $t-$ distribution, the log-likelihood contributions are of the form:

$$l_t = \frac{1}{2}\log\left[\frac{\pi(v-2)\Gamma(^{v}/_{2})^2}{\Gamma\left(^{(v+1)}/_{2}\right)}\right] - \frac{1}{2}\log\sigma_t^2 - \frac{(v+1)}{2}\log\left[1 + \frac{(y_t - X_t'\theta)^2}{\sigma_t^2(v-2)}\right] \tag{3.37}$$

where $\Gamma(.)$ is the gamma function. This distribution is always fat-tailed and produces a better fit than the normal distribution for most asset return series. The degree of freedom $v > 2$ controls the tail behaviour. The distribution is only well defined if $v > 2$ since the variance of a student- t with $v \leq 2$ is infinite, that is, the t −distribution approaches the normal distribution as $v \to \infty$.

(iii) Skewed Student-t Distribution (SSTD) is given by:

$$f(z;\ \mu,\sigma,v,\lambda) = \begin{cases} bc\left(1+\dfrac{1}{v-2}\left(\dfrac{b\left(\frac{z-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, \text{if } z < -\dfrac{a}{b} \\ bc\left(1+\dfrac{1}{v-2}\left(\dfrac{b\left(\frac{z-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, \text{if } z \geq -\dfrac{a}{b} \end{cases} \tag{3.38}$$

where v is the shape parameter with $2 < v < \infty$ and λ is the skewness parameter with $-1 < \lambda < 1$. The constants a, b and c are given as:

$$a = 4\lambda c\left(\frac{v-2}{v-1}\right), \qquad b = 1 + 3\lambda^2 - a^2, \qquad c = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)\Gamma(\frac{v}{2})}},$$

where μ and σ^2 are the mean and variance of the skewed student-t distribution respectively.

(iv) The Generalized Error Distribution (GED) is given as:

$$f(z;\ \mu,\sigma,v) = \frac{\sigma^{-1} v e^{\left(-0.5\left|\left(\frac{z-\mu}{\sigma}\right)/\lambda\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma(\frac{1}{v})}, \qquad 1 < z < \infty \tag{3.39}$$

where $v > 0$ is the degree of freedom or tail-thickness parameter and

$$\lambda = \sqrt{2^{(-2/v)}\,\Gamma\left(\frac{1}{v}\right)/\Gamma\left(\frac{3}{v}\right)}$$

If $v = 2$ the GED yields a normal distribution, if $v < 2$ the density function has thicker or fat-tails than the normal density function, whereas for $v > 2$ is has thinner tails. In order for this distribution to be used for estimating GARCH parameters, it is necessary that $v \geq 1$ since the variance is infinite when $v < 1$.

(v) Skewed Generalized Error Distribution (SGED) is given as:

$$f(z; v, \xi) = v\left(\frac{1}{2\theta\Gamma\left(\frac{1}{v}\right)}\right)\exp\left(\frac{|z-\delta|^v}{[1 - sign(z-\delta)\xi]^v\theta^v}\right) \tag{3.40}$$

where

$$\theta = \Gamma\left(\frac{1}{v}\right)^{0.5}\Gamma\left(\frac{3}{v}\right)^{-0.5} \mathrm{S}(\xi)^{-1},$$
$$\delta = 2\xi A \mathrm{S}(\xi)^{-1},$$
$$\mathrm{S}(\xi) = \sqrt{1 + 3\xi^2 - 4A^2\xi^2},$$
$$A = \Gamma\left(\frac{2}{v}\right)\boldsymbol{\Gamma}\left(\frac{1}{v}\right)^{-0.5}\Gamma\left(\frac{3}{v}\right)^{-0.5}$$

where $v > 0$ is the shape parameter controlling the height and heavy-tail of the density function while ξ is a skewness parameter of the density with $-1 < \xi < 1$.

3.9 Volatility mean reversion and half- life

For a stationary GARCH (p, q) model, the mean reverting rate is given as $(\sum \alpha_i + \beta_i)$. The magnitude of $(\sum \alpha_i + \beta_i)$ controls the speed of mean reversion. The half-life of a volatility shock is given by the formula:

$$L_{Half} = 1 - \left\{ \frac{\log(2)}{\log(\sum \alpha_i + \beta_i)} \right\} \tag{3.41}$$

The half-life of volatility represents the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it [41].

4 Results and Discussion

4.1 Summary statistics of daily log returns

We compute the summary statistics for both asset and volume of trade daily log returns and present the results in Table 1.

Table 1. Summary statistics of log returns

Statistic	Daily Stock Returns	Daily VOT Returns
Mean	0.0291717	0.0650497
Median	-0.000208	0.962818
Maximum	11.26500	524.021
Minimum	-12.5494	-506.906
Standard Deviation	1.00479	73.6711
Coefficient of Variation	34.4438	1132.54
Skewness	-0.11289	0.05388
Kurtosis	15.1179	9.12616
Jarque-Bera	28920	7386.29
P-value	0.0000	0.0000
No. of Observations	4725	4725

The descriptive statistics of log returns reported in Table 1 shows the average daily returns of 0.0292% for asset returns and 0.065% for volume of trade returns with the daily standard deviations of 1.005% and 73.671% respectively. These values reflect high levels of dispersion from the average daily returns especially for volume of trade returns in the market over the period under review. The log returns series exhibit high kurtosis values of 15.1179 and 9.12616 for asset and volume of trade respectively suggesting that big shocks of either signs are more likely to be present in the series and that the log returns series are clearly leptokurtic. The skewness coefficients are -0.11289 and 0.05388 for asset returns and volume of trade returns respectively. Zero skewness coefficients indicate evidence of lack of asymmetry, while positive or negative skewness indicates asymmetry in returns series. The negative skewness of asset returns indicate that its distribution has fat left tails, meaning that small negative movements of stock prices are not likely to be followed by equally small positive movements.

The null hypotheses of zero skewness and kurtosis coefficient of 3 are rejected at 5% significance levels suggesting that the daily returns series of asset and volume of trade in the Nigerian stock market do not follow normal distributions. This rejection of normality in the returns series is confirmed by Jarque-Bera test as its associated p-values are far below 5% significance levels. Non-Gaussianity (non-normality) is one of the *stylised facts* prominent in all financial time series data.

4.2 Graphical examination of daily stock prices and returns

The original series (daily stock prices and volume of trade values) are plotted against time and the graphical properties of the series were observed. The plots are presented in Figs. 1 and 2 respectively.

The daily stock prices and daily volume of trade values presented in Figs. 1 and 2 suggest that the series have means and variances that change with time and the presence of a trend indicating that the series are not covariance stationary. The plots of the daily stock return and volume of trade return series presented in Figs. 3 and 4 suggest that the series have constant means and variances with absence of trend indicating that they are generated by random walks and are thus weakly stationary. The plots in Figs. 3 and 4 also indicate that some periods are more clustered than others as large changes in the returns tend to be followed by large changes and small changes are followed by small changes. This phenomenon is described as *volatility clustering*.

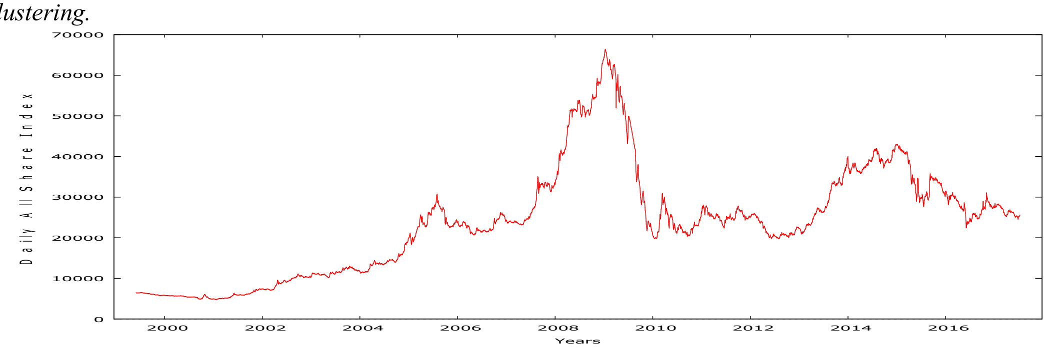

Fig. 1. Time plot of daily stock prices (ASI) from 1998 – 2017

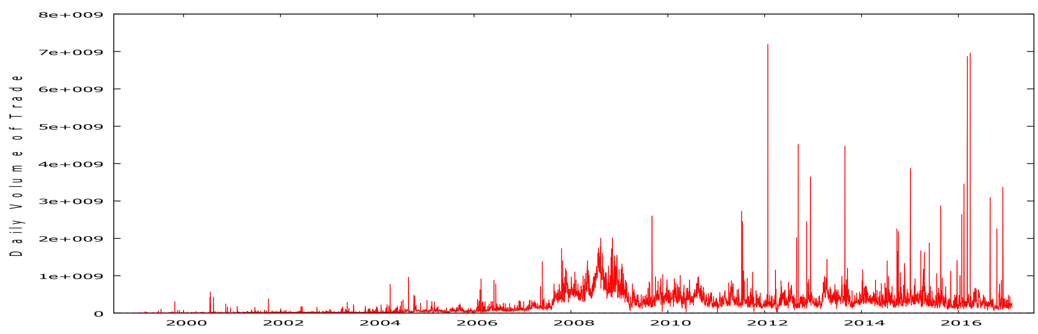

Fig. 2. Time plot of daily volume of trade values in Nigeria from 1998-2017

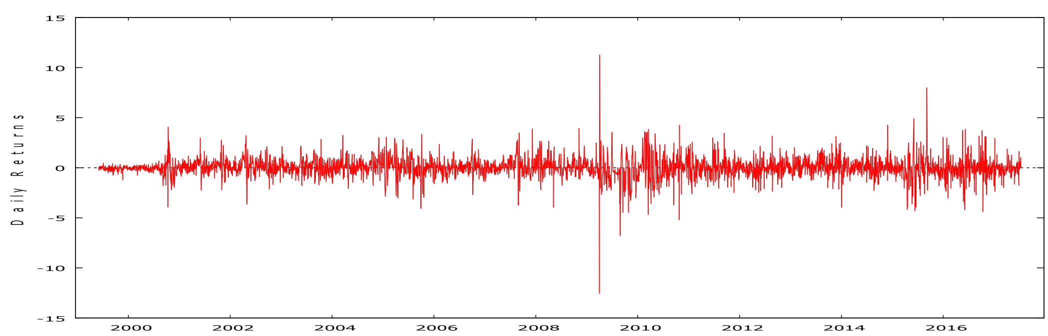

Fig. 3. Time plot of daily log returns in Nigeria

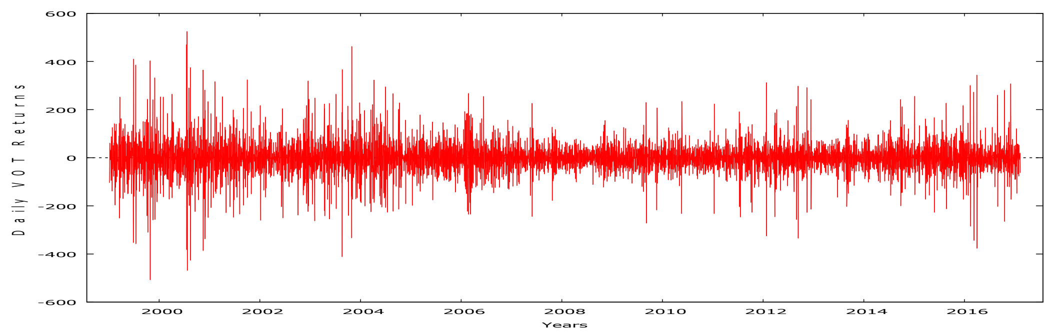

Fig. 4. Time plot of daily volume of trade log returns in Nigeria from 1998-2017

4.3 Unit root and heteroskedasticity tests results

Dickey-Fuller generalized least squares (DF GLS) unit root test is employed in examining stationarity characteristics of both daily asset prices and volume of trade values together with their daily log returns in this work. The DF GLS unit root test result is presented in Table 2 while the heteroskedasticity test for ARCH effects is presented in Table 3.

Table 2. DF GLS unit root test results

Variable	Option	Test Statistic	5% Critical Values
Dickey-Fuller Generalized Least Squares Unit Root Test Results			
Daily ASI	Intercept only	-0.4437	-1.9409
	Intercept & trend	-1.2620	-2.8900
Daily ASI Returns	Intercept only	-33.112**	-1.9409
	Intercept & trend	-33.792**	-2.8900
Daily VOT	Intercept only	-1.0826	-1.9409
	Intercept & trend	-1.7028	-2.8900
Daily VOT Returns	Intercept only	-8.2898**	-1.9409
	Intercept & trend	-12.171**	-2.8900

Note: *** denotes the significant of DF GLS test statistic at 5% significance level*

From the DF GLS unit root test results presented in Table 2, we fail to reject the null hypothesis of unit root in the daily all share index and daily volume of trade level series at 5% significance levels both with intercept only and with intercept and linear trend. However, the null hypothesis of unit root is rejected in all the log returns at 5% significance levels both with constant only and with constant and linear trend. This means that the daily all share index and volume of trade level series are non-stationary (contain unit roots) while their log returns are stationary. It is therefore worth concluding that the daily all share index and volume of trade series are all integrated of order one, I(1).

Table 3. Engle's LM heteroskedasticity test for ARCH effects

Variable	Lag	F-statistic	P-value	nR^2	P-value
Daily ASI Returns	1	1306.912	0.0000	1023.994	0.0000
	10	167.6552	0.0000	1238.844	0.0000
	20	83.8789	0.0000	1240.662	0.0000
	30	55.7513	0.0000	1239.184	0.0000
Daily VOT Returns	1	43.1105	0.0000	42.7383	0.0000
	10	11.5626	0.0000	113.1135	0.0000
	20	9.5363	0.0000	184.0799	0.0000
	30	6.9260	0.0000	200.2340	0.0000

The Engle's LM ARCH test results presented in Table 3 shows that the log returns series of asset and volume of trade both exhibit the presence of ARCH effects up to lag 30 corresponding to 4 trading weeks since the p-values of both F-statistics and nR^2 are strictly less than 0.05 significance levels. This means that the variances of these returns series are non-constant (heteroskedastic) and can only be modeled using heteroskedastic models such as symmetric and asymmetric GARCH models.

4.4 Searching for optimal symmetric and asymmetric GARCH (p, q) models

We first search for optimal symmetric and asymmetric GARCH-type models that best model the asset and volume of trade log returns series in Nigerian stock market. We use the log-likelihood in conjunction with some selected information criteria such as AIC, SIC and HQC in the presence of five error distributions namely: Normal Distribution (ND), Student-t Distribution (STD), Skewed Student-t Distribution (SSTD), Generalized Error Distribution (GED) and Skewed Generalized Error Distribution (SGED). The model with the highest log-likelihood and lowest information criteria is selected as the best fitting model for each return series. The results of the model order selection are reported in Table 4.

Table 4. Symmetric and asymmetric GARCH (p,q) models order selection for log returns

Model	Distribution	LogL	AIC	SIC	HQC
		Asset Returns			
GARCH (3,2)	STD	-5365	2.2745	2.2855	2.2784
GARCH (2,1)-M	SSTD	-5335	2.2703	2.8113	2.2760
EGARCH (2,2)	STD	-5381	2.2818	2.2942	2.2863
TGARCH (3,2)	STD	-5363	2.2727	2.2862	2.2782
PGARCH (2,3)	STD	-5376	2.2792	2.2915	2.2835
		Volume of Trade Returns			
GARCH (2,2)	STD	-26071	11.0457	11.0542	11.0459
GARCH (2,1)-M	STD	-26033	11.0297	11.0392	11.0330
EGARCH (2,1)	STD	-26012	11.0212	11.0314	11.0252
TGARCH (1,3)	STD	-26057	11.0403	11.0513	11.0442
PGARCH (3,2)	STD	-26029	11.0294	11.0436	11.0347

From the results of symmetric and asymmetric GARCH model order selection presented in Table 4, two sets of symmetric and three sets of asymmetric GARCH models are considered. The optimal selected symmetric models are GARCH (3,2) model with STD and GARCH-M (2,1) model with SSTD for asset returns and GARCH (2,2) as well as GARCH-M (2,1) models all with STD for volume of trade returns. The optimal selected asymmetric GARCH models in terms of largest log likelihoods and smallest information criteria are EGARCH (2,2), TGARCH (3,2) and PGARCH (2,3) models all with STD for asset returns and EGARCH (2,1), TGARCH (1,3) and PGARCH (3,2) all with STD for volume of trade returns. These results clearly show that heavy-tailed distributions are better at describing conditional volatility of returns in Nigerian stock market.

4.5 Parameter estimation of symmetric and asymmetric GARCH models without breaks

The results of parameter estimates of the selected GARCH-type models without breaks for asset returns are presented in Table 5 while those for volume of trade are presented in Table 6.

From the results of symmetric GARCH (3,2) and GARCH-M (2,1) models for asset returns as well as GARCH (2,2) and GARCH-M (2,1) models for volume of trade returns reported in Tables 5 and 6, all the parameters in the conditional variance equations are statistically significant. The persistence of volatility is very high in all the returns. The mean reverting rates of volatility shocks are all non-stationary as the sums of ARCH and GARCH terms ($\sum_{i=1}^{q}\alpha_i + \sum_{i=1}^{p}\beta_i$) are greater than unity in all the two returns. This shows that the conditional volatilities are unstable which can eventually explode to infinity. The non-stationarity of these symmetric GARCH models could be as a result of the fact that the returns series are contaminated with structural breaks [10,11].

Table 5. Results of symmetric and asymmetric GARCH (p, q) models for asset returns without breaks

	GARCH (3,2)	GARCH (2,1)-M	EGARCH (2,2)	TGARCH (3,2)	PGARCH (2,3)
μ	-0.0118 (0.0073)	-0.0445 (0.0168)	-0.0117 (0.0077)	-0.0114 (0.0072)	-0.0141 (0.0069)
λ	----	0.0649 (0.0290)	----	----	----
ω	0.4357 (0.0411)	0.0027 (0.0006)	0.0536 (0.0129)	7.90E-05 (1.93E-05)	0.0003 (7.93E-05)
α_1	0.4489 (0.0403)	0.4253 (0.0316)	0.3736 (0.0372)	0.3534 (0.0016)	0.3407 (0.0224)
α_2	0.0675 (0.0773)	0.1327 (0.0304)	0.0543 (0.0053)	0.0357 (0.0009)	0.0372 (0.0222)
α_3	0.0354 (0.0412)	----	----	0.0848 (0.0021)	----
γ	----	----	0.0066 (0.0031)	-0.0007 (0.0003)	-0.0012 (0.0008)
β_1	0.5019 (0.0289)	0.5106 (0.0063)	0.6654 (0.0581)	0.6760 (0.0142)	0.3385 (0.0681)
β_2	0.1528 (0.0288)	----	0.0337 (0.0057)	0.0780 (0.0139)	0.0887 (0.0124)
β_3	----	----	----	----	0.2527 (0.0592)
δ	----	----	----	----	0.8945 (0.0695)
v	5.8243 (0.4002)	4.8731 (0.3062)	5.5135 (0.3672)	5.7174 (0.3760)	5.5119 (0.3733)
φ	1.2065	1.0686	1.1270	1.2278	1.0578

Note: $\varphi = \sum_{i=1}^{q}\alpha_i + \sum_{i=1}^{p}\beta_i$ *for GARCH, GARCH-M, EGARCH and PGARCH and* $\psi = (\sum_{i=1}^{q}\alpha_i + \sum_{i=1}^{p}\beta_i + \gamma/2)$ *for TGARCH measures the shock persistence to volatility*

Table 6. Results of symmetric and asymmetric GARCH (p, q) models for volume of trade returns without breaks

	GARCH (2,2)	GARCH (2,1)-M	EGARCH (2,1)	TGARCH (1,3)	PGARCH (3,2)
μ	0.0518 (0.6804)	0.4061 (0.0911)	2.0751 (0.6946)	3.7744 (0.7423)	4.1190 (0.7306)
λ	-----	0.0579 (0.0465)	----	----	----
ω	0.0499 (0.0128)	0.6184 (0.0925)	0.1244 (0.0389)	0.0423 (0.0033)	1.9528 (0.5815)
α_1	0.5056 (0.0485)	0.5146 (0.0524)	0.2497 (0.0457)	0.4662 (0.1352)	0.3915 (0.0271)
α_2	0.0418 (0.0477)	0.0488 (0.0515)	0.1134 (0.0207)	----	0.0394 (0.0046)
α_3	----	----	----	----	0.0288 (0.0039)
γ	----	----	0.6262 (0.0443)	-0.9207 (0.1343)	-0.5103 (0.0504)
β_1	0.4023 (0.0295)	0.5109 (0.0055)	0.7759 (0.0052)	0.5883 (0.0288)	0.5238 (0.0577)

β_2	0.0577 (0.0282)	----	----	0.1656 (0.0222)	0.0433 (0.0056)
β_3	----	----	----	0.1494 (0.0207)	----
δ	----	----	----	----	0.9985 (0.0556)
v	4.5411 (0.2874)	4.2841 (0.2583)	4.3546 (0.2659)	4.0219 (0.2305)	4.2755 (0.2619)
φ	1.0074	1.0743	1.1372	0.9092	1.0268

Note: $\varphi = \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ *for GARCH, GARCH-M, EGARCH and PGARCH and* $\psi = (\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i + \gamma/2)$ *for TGARCH measures the shock persistence to volatility*

The estimated risk premium coefficients (λ) in the GARCH-M (2,1) models are positive for the two returns, which indicate that the means of the return sequence depend on past innovations and the past conditional variance. This shows that the conditional variances used as proxies for risk of returns are positively related to the levels of returns. These results show that as volatility increases, the returns correspondingly increases with a factor of 0.0649 for asset returns and 0.0579 for volume of trade returns. These results are consistent with the theory of a positive risk premium on stock indices which states that higher returns are expected for assets with higher level of risks.

From results of the asymmetric EGARCH (2,2), TGARCH (3,2) and PGARCH (2,3) models for asset returns as well as asymmetric EGARCH (2,1), TGARCH (1,3) and PGARCH (3,2) models for volume of trade return series presented in Tables 5 and 6, we observe that all the parameters of the models in the variance equations are statistically significant at 5% significance levels. The impacts of shocks on conditional volatility are asymmetric with the absence of leverage effects. The coefficients of the asymmetric and leverage effect parameters (γ) are positive and statistically significant for EGARCH models but negative and statistically significant for TGARCH and PGARCH models in both returns. This indicates that market retreats (bad news) produce less volatility than market advances (good news) of the same magnitude in the Nigerian stock market. However, the conditional variance processes of the two EGARCH and two PGARCH models for asset and volume of trade returns respectively seem to be unstable and non-stationary as the sums of ARCH and GARCH terms are greater than unity (i.e., $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i > 1$). Also, the TGARCH (3,2) model for asset return is unstable and non-stationary since $(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i + \gamma/2) = 1.2278 > 1$. For the volume of trade returns, the TGARCH (1,3) model is stable and stationary as $(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i + \gamma/2) = 0.9092 < 1$. This shows that the conditional volatility for volume of trade in Nigerian stock market is mean reverting as well as predictable. From the estimated symmetric and asymmetric GARCH-type models, both asset and volume of trade returns retain the fat tails behaviour typical of financial time series data since the shape parameters (v) for both student-t and skewed student-t distributions are all greater than two.

From the estimated models, we observe that the shock persistence is very high in both returns which raise the possibility that the returns are contaminated with structural breaks. When returns are contaminated with breaks, the estimates of their volatility are biased to unity, see the works of [10,11]. Hence there is every need to investigate the presence of structural breaks in the series.

4.6 Bai and Perron multiple breakpoints test results

We apply the Bai and Perron multiple breakpoints test procedure to the log return series and obtained different break points and dates for asset returns and volume of trade log returns of the Nigerian stock market. We detect a maximum of 4 break points for asset returns and a maximum of 5 break points for volume of trade returns. The structural break points in volatility with time periods are presented in Table 7.

Table 7. Structural breakpoints in volatility with time periods

Returns	Break points	Time periods

Asset Returns	4	9th June, 1999 – 6th August, 1999 30th November, 2000 – 15th January, 2001 27th December, 2007 – 30th October, 2008 23rd December, 2008 – 17th March, 2009
Volume of Trade	5	10th December, 2004 – 31th March, 2005 3rd July, 1998 – 10th December, 1998 12th August, 1999 – 28th December, 1999 23rd December, 2008 – 10th March, 2009 27th October, 2011 – 27th June, 2012 17th March, 2016 – 11th April,2016

From the results of multiple breakpoints presented in Table 7, we observe that the major reason for these structural break points is the global financial crisis which triggered in 2007 up to 2009 and had its origin in the US financial market and spread rapidly to other developed and emerging financial markets has affected the Nigerian stock market as well. The economic recession and the banking reforms in 2004 and the prices of oil problems in the country were other causes. Also in the 2005-2006, the economic recovery in Nigeria commonly affected the Nigerian stock market. The terrorist attacks of Niger Delta militants in 2011-2012 and Boko Haram attacks in 2013-2016 were also a contributing factor. The other breaks detected are as a result of internal, local, domestic, political or economic crises in the country.

4.7 Parameter estimates of symmetric and asymmetric volatility models with breaks

We now incorporate the detected structural breaks in form of indicator (dummy) variable into the volatility models. The parameter estimates of the models for asset returns are presented in Table 8 while those for volume of trade returns are reported in Table 9.

Table 8. Results of symmetric and asymmetric GARCH (p, q) models for asset returns with breaks

	GARCH (3,2)	GARCH (2,1)-M	EGARCH (2,2)	TGARCH (3,2)	PGARCH (2,3)
μ	0.0092 (0.0074)	-0.0500 (0.0170)	-0.0093 (0.0078)	-0.0088 (0.0073)	-0.0086 (0.0074)
λ	----	0.0805 (0.0299)	----	----	----
ϕ	-0.1985 (0.0439)	-0.1517 (0.0412)	-0.2303 (0.0438)	-0.1928 (0.0465)	-0.2079 (0.0461)
ω	0.0005 (0.0001)	0.0028 (0.0006)	-0.0585 (0.0140)	0.0008 (0.0002)	0.0002 (4.73E-05)
α_1	0.4509 (0.0409)	0.3237 (0.0316)	0.5767 (0.0372)	0.2263 (0.0021)	0.4239 (0.0352)
α_2	0.0827 (0.0782)	0.1295 (0.0304)	0.0078 (0.0034)	0.0712 (0.0023)	0.1212 (0.0350)
α_3	0.0326 (0.0417)	----	----	0.1470 (0.0004)	----
γ	----	----	0.5013 (0.0351)	-0.0009 (0.0003)	-0.0009 (0.0004)
β_1	0.2436 (0.0295)	0.5087 (0.0063)	0.2642 (0.0099)	0.2622 (0.0157)	0.2544 (0.0127)
β_2	0.0444 (0.0294)	----	0.0610 (0.0058)	0.1639 (0.0153)	0.0082 (0.0014)
β_3	----	----	----	----	0.1487 (0.0303)
δ	----	----	----	----	1.7184 (0.1782)
υ	5.6883	4.8104	5.3462	5.6255	5.5857

	(0.3821)	(0.3319)	(0.3453)	(0.3598)	(0.3788)
φ	0.8542	0.9619	0.9097	0.8661	0.9564

Note: *$\varphi = \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ for GARCH, GARCH-M, EGARCH and PGARCH and $\psi = (\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i + \gamma/2)$ for TGARCG measures the shock persistence to volatility*

The estimated results of symmetric GARCH (3,2) and GARCH (2,2) models with breaks for asset and volume of trade returns as well as GARCH-M (2,1) models for both asset and volume of trade returns reported in Tables 8 and 9 show significant decrease in the persistence parameters β_i for all the estimated models as a result of incorporating the sudden level shifts in the conditional variances of the models. The mean reverting rates $(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i)$ also declined significantly for all the stock returns as a result of including these level shifts in the conditional variance equations. All the estimated parameters in the variance equations of the models are statistically significant at the 5% significance levels. The estimated risk premium coefficients (λ) in the GARCH (2,1)-M models are also positive for both asset and volume of trade returns indicating that the conditional variances used as proxies for risk of returns are positively related to the levels of returns. This result corroborates the empirical findings of several authors [42,43,25,44,45] but contrary to the findings of several authors [46,47,48,49].

Table 9. Results of symmetric and asymmetric GARCH (p, q) models for volume of trade returns with breaks

	GARCH (2,2)	GARCH (2,1)-M	EGARCH (2,1)	TGARCH (1,3)	PGARCH (3,2)
μ	-0.0143	0.3986	2.1098	3.7466	4.4593
	(0.7291)	(0.0445)	(0.7159)	(0.7743)	(0.7345)
λ	-----	0.4610	----	----	----
		(0.0468)			
ϕ	-0.1233	-0.1170	-0.4507	-0.3301	-1.3073
	(0.0265)	(0.0208)	(0.0202)	(0.0228)	(0.1003)
ω	0.4113	0.1809	0.1254	0.9039	2.0579
	(0.0299)	(0.0449)	(0.0391)	(0.0486)	(0.6092)
α_1	0.3178	0.2644	0.3243	0.3068	0.3901
	(0.0606)	(0.0524)	(0.0257)	(0.0135)	(0.0271)
α_2	0.2124	0.1483	0.1138	----	0.0513
	(0.0593)	(0.0515)	(0.0042)		(0.0504)
α_3	----	----	----	----	0.0892
					(0.0281)
γ	----	----	0.6257	-0.5208	-0.2397
			(0.0443)	(0.0352)	(0.0294)
β_1	0.3443	0.5805	0.5017	0.3883	0.2069
	(0.0314)	(0.0056)	(0.0052)	(0.0288)	(0.0576)
β_2	0.0449	----	----	0.2605	0.1413
	(0.0289)			(0.0222)	(0.0559)
β_3	----	----	----	0.1495	----
				(0.0127)	
δ	----	----	----	----	1.0025
					(0.0551)
v	3.9508	4.2826	4.3549	4.0220	4.2895
	(0.2286)	(0.2586)	(0.2661)	(0.2306)	(0.2643)
φ	0.9194	0.9932	0.9398	0.6447	0.8795

Note: *$\varphi = \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ for GARCH, GARCH-M, EGARCH and PGARCH and $\psi = (\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i + \gamma/2)$ for TGARCH measures the shock persistence to volatility*

For the asymmetric EGARCH (2,2), TGARCH (3,2) and PGARCH (2,3) models for asset returns as well as EGARCH (2,1), TGARCH (1,3) and PGARCH (3,2) models for volume of trade returns reported in Tables 8 and 9, we observe also that by incorporating the structural break points in the volatility models, there are significant decreases in the values of shock persistence parameters (β_i) in all the estimated asymmetric GARCH-type models. There are also significant reductions in the values of the mean reversion rates ($\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$) in all the estimated models of the stock market returns. Also by including the structural breaks in these models, the stationarity and stability conditions of the models are satisfied as the sum of ARCH and GARCH terms are less than one in all the estimated asymmetric GARCH models with breaks. This shows that the conditional variance process is stable and predictable and that the memories of volatility shocks are remembered in Nigerian stock market when structural breaks are incorporated in volatility models. All the estimated asymmetric models retain the asymmetric response property without the presence of leverage effects. These findings corroborate the empirical findings of several authors [9,15,17,19,21] among others.

The coefficients of the dummy variable (ϕ) is negative and statistically significant in all the estimated symmetric and asymmetric GARCH models suggesting that the global financial crisis, the economic recession, the crude oil price fluctuations, the Niger Delta militant and Boko Haram terrorism which contaminated the asset and volume of trade return series have negatively affected the Nigerian stock market during the study period.

4.8 The magnitude of news impact on the conditional variance

To measure the impact of bad or good news on the conditional volatility in the Nigerian stock market, we consider both the models with and without breaks for both asset and volume of trade returns. Results are presented in Table 10.

Table 10. The magnitude of news impact on conditional volatility of estimated models

	Models without Breaks			
Model	**Asset Returns**		**Volume of Trade Returns**	
	Bad news	**Good news**	**Bad news**	**Good news**
EGARCH	0.9934	1.0066	0.3738	1.6262
TGARCH	0.4732	0.4739	0.4545	0.4662
PGARCH	0.3767	0.3791	0.0506	0.6100
	Models with Breaks			
EGARCH	0.4987	1.5013	0.3743	1.6257
TGARCH	0.4436	0.4445	0.2140	0.3068
PGARCH	0.5442	0.5460	0.2509	0.7703

Note: *Asymmetry is calculated as* $\frac{|-1+\hat{\gamma}|}{1+\hat{\gamma}}$ *for EGARCH,* $\frac{\sum \hat{\alpha}_i+\hat{\gamma}}{\sum \hat{\alpha}_i}$ *for TGARCH and* $\frac{|\sum \hat{\alpha}_i+\hat{\gamma}|}{|\sum \hat{\alpha}_i-\hat{\gamma}|}$ *for PGARCH models.*

The evidence provided by Table 10 shows that good news have more impact on conditional volatility than bad news for all the estimated asymmetric EGARCH, TGARCH and PGARCH models for both asset and volume of trade returns under heavy tailed student-t error distributions. In the EGARCH models for instance, the impact of good news on conditional volatility is about 1.0066 times more than bad news for asset returns and about 1.6262 times more than bad news for volume of trade returns for models without breaks. For the asymmetric EGARCH models with breaks, the impact of good news is about 1.5013 times more than bad news for asset returns and about 1.6257 times more than bad news for volume of trade returns. This clearly shows the absence of leverage effect in Nigerian stock market.

4.9 Models comparison in terms of volatility half-life

We compared the models without breaks with those with breaks in terms of volatility half-lives for both asset and volume of trade returns. According to Engle and Bollerslev [41], volatility half-life represents the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it. The results are presented in Table 11.

From the results of models comparison reported in Table 11, we observe that the GARCH family models estimated without incorporating the sudden shifts in variance are non-stationary, unstable and unpredictable as the sum of ARCH and GARCH terms are more than unity apart from TGARCH (1,3) estimated for volume of trade returns. The volatility shock persistence of the models are very high giving rise to long memory in the conditional variance process. The volatility half-lives of these models also explode to infinity and do not mean revert to the long-run average volatility levels.

However, by incorporating the structural break points in the volatility models, we observe significant reductions in the values of mean reversion rates $(\alpha_i + \beta_i)$ in all the estimated models of the stock market returns. Also by accounting for the structural breaks in these models, the volatility half-lives of these models also mean revert to the long-run average volatility levels.

Table 11. Comparison of estimated models with and without breaks and volatility half-lives

Model	Without level Shifts		With level Shifts	
	Shock Persistence	Half-life (days)	Shock Persistence	Half-life (days)
		Asset Returns		
GARCH (3,2)	1.2065	∞	0.8542	4
GARCH (2,1)-M	1.0686	∞	0.9619	19
EGARCH (2,2)	1.1270	∞	0.9097	7
TGARCH (3,2)	1.2278	∞	0.8661	5
PGARCH (2,3)	1.0578	∞	0.9564	16
		Volume of Trade Returns		
GARCH (2,2)	1.0074	∞	0.9194	8
GARCH (2,1)-M	1.0743	∞	0.9932	102
EGARCH (2,1)	1.1372	∞	0.9398	11
TGARCH (1,3)	0.9092	7	0.6447	2
PGARCH (3,2)	1.0268	∞	0.8795	5

Note: *volatility half-life is computed as:* $\boldsymbol{L}_{half} = 1 - \{log(2)/log(\sum \alpha_i + \beta_i)\}$

4.10 Post-estimation test for ARCH effects and serial correlation

To test for the remaining ARCH effects as well as serial correlation in the residuals of returns for the estimated GARCH models, we employ Engle's LM test and Ljung-Box Q-statistic test. The results are presented in Table 12.

Table 12. ARCH LM test and Q-statistics tests of residuals of estimated GARCH-type models

Model	Lag	ARCH LM Test		Squared Residuals	
		F-statistic	P-value	Q-statistic	P-value
		Asset Returns without Breaks			
GARCH (3,2)	31	0.2149	0.9987	6.8114	1.0000
GARCH (2,1)-M	31	0.3142	0.9999	9.6208	0.9996
EGARCH (2,2)	31	0.2348	0.9969	7.4756	0.9999
TGARCH (3,2)	31	0.2551	0.9998	6.9439	1.0000
PGARCH (2,3)	31	0.2918	0.9993	6.5452	0.9999
		Asset Returns with Breaks			
GARCH (3,2)	31	0.0862	0.7690	5.2607	1.0000
GARCH (2,1)-M	31	0.1609	0.6883	8.6658	1.0000
EGARCH (2,2)	31	0.2403	0.6242	7.1756	0.9998
TGARCH (3,2)	31	0.0679	0.7944	5.7720	1.0000
PGARCH (2,3)	31	0.0414	0.8388	5.9376	0.9898
		Volume of Trade Returns without Breaks			

GARCH (2,2)	31	1.3520	0.8658	41.6672	0.9605
GARCH (2,1)-M	31	1.3935	0.2483	30.9467	0.2518
EGARCH (2,1)	31	2.7636	0.2798	36.8573	0.2163
TGARCH (1,3)	31	0.8804	0.5509	21.1136	0.1749
PGARCH (3,2)	31	1.1329	0.2800	35.3832	0.2693
Volume of Trade Returns with Breaks					
GARCH (2,2)	31	0.9655	0.7980	21.2741	0.6819
GARCH (2,1)-M	31	2.6593	0.1030	18.8604	0.9374
EGARCH (2,1)	31	2.7424	0.0978	36.8271	0.2554
TGARCH (1,3)	31	0.1588	0.6903	31.6961	0.6448
PGARCH (3,2)	31	2.9517	0.7832	28.8299	0.8993

The results of the ARCH LM tests of residuals for the remaining ARCH effects of the estimated GARCH-type models with and without structural breaks shown in Table 12 indicates that the conditional variance equations for the five GARCH family models have captured all the ARCH effects in the residuals of the stock return series and none is left as the p-values of the F-statistics tests associated with the ARCH LM tests are highly statistically insignificant. This means that the GARCH models sufficiently captured all the ARCH effects in the residuals of both returns. The results of the Ljung-Box Q-statistics tests also presented in Table 12 shows the absence of serial correlation in the residuals of both returns as its associated p-values are highly statistically insignificant. This shows that the residuals of the estimated GARCH-type models are purely random processes and the models fits are good.

5 Conclusion and Recommendations

This study investigated the impact of volatility shock persistence on the conditional variance in the Nigerian stock returns using symmetric and asymmetric higher order GARCH family models in the presence of random level shifts and heavy-tailed distributions. The study employed Bai and Perron methodology to detect structural breakpoints in the conditional variance of daily stock and volume of trade returns in Nigerian stock market from 2nd January, 1998 to 22nd March, 2017. Two sets of symmetric GARCH models i.e., GARCH (3,2) and GARCH (2,1)-M for asset returns, GARCH (2,2) and GARCH (2,1)-M for volume of trade returns and three sets of asymmetric GARCH models i.e., EGARCH (2,2), TGARCH (3,2) and PGARCH (2,3) for asset returns as well as EGARCH (2,1), TGARCH (1,3) and PGARCH (3,2) models for volume of trade returns were optimally selected using information criteria and log likelihood as the best fitting symmetric and asymmetric GARCH models to estimate conditional volatility with and without structural breaks.

From the estimated results, it was found that when random level shifts were ignored in volatility models, the shocks persistence were very high with long memory and variance explosion. But when models were estimated with dummy variables for the detected structural breaks, there was a significant reduction in shocks persistence and long memory. Moreover, volatility half-lives also declined drastically while accounting for these sudden level shifts in variance. The study found asymmetry with no leverage effects as well as a positive risk-return tradeoff for both asset and volume of trade returns in the Nigerian stock market. The findings of this study are very crucial and informative to both investors and traders who might want to invest in Nigerian stocks as well as policy makers in Nigerian stock market and Nigerian stock exchange since structural breaks caused by financial and economic crises can affect investors' decision in a stock market.

The study therefore recommends that, policy makers should take into account these regime changes in their financial policy design; volatility estimation in the Nigerian stock market using symmetric and asymmetric GARCH models should incorporate dummy variables in the conditional variance of returns and non-normal distributions to avoid over-estimation of volatility shocks; investors in the Nigerian stock market should be compensated for holding risky assets; excessive and more aggressive trading strategy for all securities will increase market depth and consequently reduce the volatile nature of the Nigerian stock market.

Competing Interests

Authors have declared that no competing interests exist.

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