



Stability Analysis of Competition among Companies: A Guide from Predator Prey Modeling

Titus Ifeanyi Chinebu^{1*}, Nnaoma Ugenyi² and Edmund Onwubiko Ezennorom¹

¹Department of Computer Science, Madonna University Nigeria, Elele, Nigeria.

²Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper studied the behavior of two companies using predator prey model as the basis. As the companies are competing constantly, it affects them because their interaction determines the availability of resources for their growth. Considering growth of these companies, the parameters k and γ which were respectively the carrying capacity and competitive impact of either of the competing companies on each other were included in the model. Equilibrium point and their existence criteria were analyzed to find the threshold that will guarantee the coexistence of both companies or collapse of either of them or both. It was shown that both companies can grow and rise simultaneously, (coexist) by dividing their resources correspondingly or that even the slightest change in their competition coefficient can lead to adverse situation, which may cause complete disappearance of one of the companies or both. We conclude that as long as these companies did not operate beyond their effective carrying capacity and equally maintain their respective competitive advantage, coexistence might be achieved. Some simulations are also given to illustrate our results.

Keywords: Predator prey model; competition; competitive advantage; competitive coefficient; carrying capacity; resources.

*Corresponding author: E-mail: chinebutitus@yahoo.com;

1 Introduction

The two species model by Lotka – Volterra in 1920's is the simplest form of mathematical model for predator prey dynamics and it forms the basis of many models used in the analysis of population dynamics [1]. Indeed Lotka – Volterra model has been extended to describe competition beyond biology and ecology. It has opened way to efficiently managing competition in the market place [2]. An intriguing aspect of the market place is that the nature of competition can change over time. A technology, company or product does not need to remain prey to another forever. This simply implies that competition role can be radically altered with technological advancements or with the right marketing decisions. Also, for most companies today, the only truly sustainable advantage comes from out innovating their competitors [3].

Actually, some large companies not only want to be successful in their competition, but they also want to have complete control or monopolize in the market place. They make efforts to drive other companies (competitors) out of business or even simply purchase them to take them out of competition [4]. One method is the hostile takeover, where a company or even wealthy individuals will purchase enough stock in a company in order to take control of it. They may then dismantle the company and sell it off; thus eliminating a competitor.

We shall be looking at the type of competition where a stronger company makes an unprovoked attack on a weaker company. Here the stronger company is taken as the predator while the weaker company is regarded as the prey. In some cases, it isn't really a competition because the result of the attack is a quick victory. Other times, the attacker gains a competitive advantage that can lead to victory. The weaker company may try to flee and sometimes will go on the defense. On some occasions, the predator (stronger company) may over estimate the ease of victory, and the victim (prey) may counter attack, turning the competition into a head-to-head battle. In this situation, both companies will continuously compete with each other without any of them winning the competition.

The purpose of most competitions is to either win the contest or gain a competitive advantage in a continuing contest. In a predator-prey competition, the opponent may be unsuspecting of the attack or at least may not have agreed to compete. Usually the predator (stronger company) that is better prepared is ready for the competition. Sometimes the prey (weaker company) may use a surprise attack to gain a competitive advantage, in anticipation of a later head-to-head competition.

Remarking that competition is a constructive force in this model, we assume that weak competition can lead the system (companies) to extinction. A stronger interaction can stabilize a fixed production. In some predatory competition, the stronger company may eventually succeed in having a high market share thereby driving the weaker company out of business once they see them as potential competitors. According to Hayes [5], market share represents the percentage of an industry or a market's total sales that is earned by a particular company over a specified time period. It is that portion of a market controlled by a particular company or product.

2 Related Literature

According to Von Arb [6], there has been little research work into the role of using predator prey model relation between specific companies, but there has been much research in the role of predator prey models in the field of economics. He further stated that the fundamental model in economics is the Godwin's model which attempts to model economic fluctuation in general by relating real wages and real employment. Cooper and Nakanishi [7] developed an interesting and relevant predator prey model into the dynamics of the Korean Stock Exchange (KSE) and the Korean Security Dealers Automated Quotation (KSDAQ), both competing in the Korean Stock Market. In their research, the KSE was regarded as the prey to KSDAQ until eventually the two markets stabilized into a pure competition relationship.

Modis [2] disclosed the struggle between fountain pens and ballpoint pens from 1929 to 2000. He stated that the substitution of ballpoint pens for the fountain pens as a writing instrument went through three distinct

stages after which fountain pens successfully secured a healthy and profitable market niche. This simply shows that there are many important applications of predator prey models which actually include the economic spectrum. Certainly, predator prey models have been used to estimate the behavior of individual companies [6]. Zelga [8] stated that competitive company has the ability and flexibility to adapt to changing conditions and make decisions that will provide a competitive advantage.

Romanov et al. [9] devoted their paper to the simulation of two population's coexistence, one of which represents old big corporations and the other – small innovative business. The most important result of their research is the discovery of parameter values interval in which great firms and small business (start ups) can grow and raise simultaneously. Suebcharoen [10] studied the behavior of a predator-prey model with switching and stage-structure to predator. Bing et al. [11] introduced a non-smooth switched harvest into a simple predator-prey model with logistic growth of the prey and a bilinear function response. They showed that when the density of the predator is below a switched value, the harvest rate is linear; otherwise, it will be constant. Sayekti [12] studied the interaction between prey, secondary predator, and primary predator using mathematical model of one prey and two predator system with constant harvesting in prey population. They found that as long as harvesting rate in prey population is smaller than prey intrinsic growth rate, coexistence might be achieved. Here, we shall provide an analytical study of a model of the interaction between two companies based on predator prey.

3 Model Formulation

In this section, the interaction between the predator x_1 and the prey x_2 will be constructed as a deterministic model in two dimension. Let us assume that the two companies (x_1, x_2) are operating together in the same market place. Each of these companies evolves subsequent to a logistic type dynamics. The competitive interaction between these companies may vary their growth rate with time and this shows that their growth rate is dependent on how successful any of them are in the competition for available resources.

According to MacArthur [1], Volterra developed the basic theory of competition, using the logistic equation as the framework. This assertion was that the 'effective carrying capacity' of the environment is reduced by the presence of competitors for resources. Suppose that two companies compete for the same resources (raw material and market), and assuming there is limited rate of supply of these resources. Different companies would have different carrying capacity. Let one of the companies be x_1 with effective carrying capacity k_1 (the predator) and is supposed to benefit by attacking the other company x_2 with effective carrying capacity k_2 (the pray) and on the contrary, the later company x_2 suffers damage as a consequence of the attack of the other company x_1 . Then both companies are governed by an attack – defense interaction which takes place between them.

If x_1 is much smaller than k_1 and x_2 much smaller than k_2 , then resources are plentiful and the companies grows exponentially with growth rate r_1 and r_2 . If company 1 (first company) and company 2 (second company) competes, depending on whoever wins, then the growth of one reduces the resources (raw material and market) available for the other and vice versa. Since we do not know the impact the first or the second company have on each other, we introduce two additional parameters to model the competition. The parameter γ represents some kind of mixed production rate containing also information on the mutual interaction due to competition between the companies. Therefore, base on the above assumptions, we proposed our two dimension predator and prey mathematical model which is a reasonable modified coupled logistic equation, given as

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1 + \gamma_{21} x_2}{k_1} \right) \quad (1)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2 + \gamma_{12} x_1}{k_2} \right) \quad (2)$$

From (1), x_2 represents the initial resources (capital, partners, suppliers, customers) available for company 2, while γ_{21} represents the impact of the second company's competition effect (competitive advantage) on the

first company (i.e., the per capita reduction in carrying capacity of the first company caused by competition with the second company). Therefore, we call γ_{21} the competition coefficient of the company 2 acting on company 1. The subscript 21, indicates the effect of company 2 on the growth of company 1.

Similarly, in equation (2) x_1 represents the initial resources (capital, partners, suppliers, customers) available for company 1 whereas γ_{12} represents the impact of the first company's competition effect (competitive advantage) on the second company (i.e., the per capita reduction in carrying capacity of the second company caused by competition with the first company). Therefore, we call γ_{12} the competition coefficient of the company 1 acting on company 2. Also, the subscript 12, indicates the effect of company 1 on the growth of company 2.

For one of the companies to win in the competition, the company must have a positive growth rate that is, $\frac{dx_1}{dt} > 0$ and $\frac{dx_2}{dt} > 0$. Since the two companies are competing with each other, we need $\frac{dx_1}{dt} > 0$ and $\frac{dx_2}{dt} > 0$ even when the operations of company x_n and y_n nears their carrying capacities k_1 and k_2 . Thus we have

$$\frac{r_1 x_1 (k_1 - x_1 - \gamma_{21} x_2)}{k_1} > 0 \quad (3)$$

$$\frac{r_2 x_2 (k_2 - x_2 - \gamma_{12} x_1)}{k_2} > 0 \quad (4)$$

From (3) we have

$$k_1 - x_1 - \gamma_{21} x_2 > 0 \quad (5)$$

$$\gamma_{21} < \frac{k_1 - x_1}{x_2} \quad (6)$$

From (4) we obtain

$$k_2 - x_2 - \gamma_{12} x_1 > 0 \quad (7)$$

$$\gamma_{12} < \frac{k_2 - x_2}{x_1} \quad (8)$$

Substituting $x_1 = 0$ and $x_2 = k_2$ into equation (6) we obtain

$$\gamma_{21} < \frac{k_1 - 0}{k_2} \Rightarrow \gamma_{21} < \frac{k_1}{k_2}$$

Also, substituting $x_2 = 0$ and $x_1 = k_1$ into equation (8) we obtain

$$\gamma_{12} < \frac{k_2 - 0}{k_1} \Rightarrow \gamma_{12} < \frac{k_2}{k_1}$$

Observe that the right-hand side of the two inequalities is reciprocal of one another. If we let $\frac{k_1}{k_2} = p$, then $\frac{k_2}{k_1} = \frac{1}{p}$. Next we find equilibria of system (1) and (2) by equating the derivatives on the left-hand side to zero, that is $\frac{dx_1}{dt} = \frac{dx_2}{dt} = 0$. The equilibria are solutions of the system.

$$\frac{r_1 x_1 (k_1 - x_1 - \gamma_{21} x_2)}{k_1} = 0 \quad (9)$$

$$\frac{r_2 x_2 (k_2 - x_2 - \gamma_{12} x_1)}{k_2} = 0 \quad (10)$$

From (9) we get

$$\begin{aligned} k_1 - x_1 - \gamma_{21} x_2 \\ x_2 = \frac{k_1 - x_1}{\gamma_{21}} \end{aligned} \quad (11)$$

From (10) we obtain

$$\begin{aligned} k_2 - x_2 - \gamma_{12} x_1 = 0 \\ x_1 = \frac{k_2 - x_2}{\gamma_{12}} \end{aligned} \quad (12)$$

Solving equations (11) and (12) simultaneously as in the appendix below, we obtain

$$x_1 = \frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} = (1 - \gamma_{21}) \frac{1}{1 - \gamma_{12}\gamma_{21}} \text{ and } x_2 = \frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} = (1 - \gamma_{12}) \frac{1}{1 - \gamma_{12}\gamma_{21}}$$

3.1 Equilibrium point analysis

Observe that equations (1) and (2) when examined simultaneously, describes the dynamics of two companies competing with each other. Since there is no explicit function that serves as solutions to these simultaneous equations, we use a variety of methods to examine their predictions. Having that $k_1, k_2, r_1, r_2, \gamma_{12}, \gamma_{21}$ are positive constants and given a range of parameter values and some initial values for x_1 and x_2 at $t = 0$, we would typically like to know if the final outcome is one of the following possibilities;

- (i) Both companies suffer from each other's existence by collapsing
- (ii) One of the companies suffer by collapsing because of the existence of the other
- (iii) Both companies coexist.

Assuming $k_1 = k_2 = 1$, we have the possible fixed points as

- (i) $(x_1^*, x_2^*) = (0, 0)$
- (ii) $(x_1^*, x_2^*) = (k_1, 0) = (1, 0)$
- (iii) $(x_1^*, x_2^*) = (0, k_2) = (0, 1)$
- (iv) $(x_1^*, x_2^*) = \frac{1}{1 - \gamma_{12}\gamma_{21}} (1 - \gamma_{21}, 1 - \gamma_{12})$

We further represent equations (1) and (2) in the form

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = G \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (13)$$

from which we find the Jacobian matrix of the model system by differentiating equations (1) and (2) with respect to x_1 and x_2 respectively to obtain

$$\frac{dx_1^*}{dt} = \left[r_1 - \frac{2r_1x_1 + r_1\gamma_{21}x_2}{k_1} \right] x_1^* + \left[-\frac{r_1\gamma_{21}x_1}{k_1} \right] x_2^*$$

$$\frac{dx_2^*}{dt} = \left[-\frac{r_2\gamma_{12}x_2}{k_2} \right] x_1^* + \left[r_2 - \frac{2r_2x_2 + r_2\gamma_{12}x_1}{k_2} \right] x_2^*$$

Remember that $k_1 = k_2 = 1$, then we have the Jacobian matrix as

$$J(G) = \begin{bmatrix} r_1(1 - 2x_1 - \gamma_{21}x_2) & -r_1\gamma_{21}x_1 \\ -r_2\gamma_{12}x_1 & r_2(1 - 2x_2 - \gamma_{12}x_1) \end{bmatrix}$$

(i) Using the fixed points, we test the stability at $(x_1^*, x_2^*) = (0, 0)$ and have the characteristic determinant as

$$|G - \lambda I| = \begin{vmatrix} r_1 - \lambda & 0 \\ 0 & r_2 - \lambda \end{vmatrix}$$

Then, the characteristic equation is $|G - \lambda I| = 0$ which gives

$$\begin{vmatrix} r_1 - \lambda & 0 \\ 0 & r_2 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (r_1 - \lambda)(r_2 - \lambda) = 0$$

$$\therefore \lambda_1 = r_1 \text{ or } \lambda_2 = r_2$$

This simply implies that $(0, 0)$ is an unstable equilibrium point.

(ii) We test the stability at $(x_1^*, x_2^*) = (0, 1)$ and have the characteristic determinant as

$$|G - \lambda I| = \begin{vmatrix} r_1(1 - \gamma_{21}) - \lambda & 0 \\ -r_2\gamma_{12} & -r_2 - \lambda \end{vmatrix}$$

Then, the characteristic equation is $|G - \lambda I| = 0$ which gives

$$\begin{vmatrix} r_1(1 - \gamma_{21}) - \lambda & 0 \\ -r_2\gamma_{12} & -r_2 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (r_1(1 - \gamma_{21}) - \lambda)(-r_2 - \lambda) = 0$$

$$\therefore \lambda_1 = r_1(1 - \gamma_{21}) \text{ or } \lambda_2 = -r_2$$

Therefore, $(0, 1)$ is asymptotically stable if and only if $\gamma_{21} > 1$ and unstable if $\gamma_{21} < 1$.

(iii) We test the stability at $(x_1^*, x_2^*) = (1, 0)$ and have the characteristic determinant as

$$|G - \lambda I| = \begin{vmatrix} -r_1 - \lambda & -r_1\gamma_{21} \\ 0 & r_2(1 - \gamma_{12}) - \lambda \end{vmatrix}$$

Then, the characteristic equation is $|G - \lambda I| = 0$ which gives

$$\begin{vmatrix} -r_1 - \lambda & -r_1\gamma_{21} \\ 0 & r_2(1 - \gamma_{12}) - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (-r_1 - \lambda)(r_2(1 - \gamma_{12}) - \lambda) = 0$$

$$\therefore \lambda_1 = -r_1 \text{ or } \lambda_2 = r_2(1 - \gamma_{12})$$

Therefore, (1, 0) is asymptotically stable if and only if $\gamma_{12} > 1$ and unstable if $\gamma_{12} < 1$.

(iv) We test the stability at $(x_1^*, x_2^*) = \frac{1}{1 - \gamma_{12}\gamma_{21}}(1 - \gamma_{21}, 1 - \gamma_{12})$ and have the characteristic determinant as

$$|G - \lambda I| = \begin{vmatrix} r_1 \left(1 - 2 \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{21} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda & -r_1\gamma_{21} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \\ -r_2\gamma_{12} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) & r_2 \left(1 - 2 \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{12} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda \end{vmatrix}$$

Then, the characteristic equation is $|G - \lambda I| = 0$ which gives

$$\begin{vmatrix} r_1 \left(1 - 2 \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{21} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda & -r_1\gamma_{21} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \\ -r_2\gamma_{12} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) & r_2 \left(1 - 2 \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{12} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left[r_1 \left(1 - 2 \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{21} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda \right] \left[r_2 \left(1 - 2 \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) - \gamma_{12} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \right) - \lambda \right] - \left[-r_1\gamma_{21} \left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) \right] \left[-r_2\gamma_{12} \left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) \right] = 0$$

If we let $\left(\frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} \right) = a$ and $\left(\frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} \right) = b$, then we obtain

$$\begin{aligned} & [r_1(1 - 2a - \gamma_{21}b) - \lambda][r_2(1 - 2b - \gamma_{12}a) - \lambda] - [r_1\gamma_{21}a][r_2\gamma_{12}b] = 0 \\ & [(r_1 - 2ar_1 - \gamma_{21}br_1) - \lambda][(r_2 - 2br_2 - \gamma_{12}ar_2) - \lambda] - [r_1r_2\gamma_{12}\gamma_{21}ab] = 0 \\ & r_1[(r_2 - 2br_2 - \gamma_{12}ar_2) - \lambda] - 2ar_1[(r_2 - 2br_2 - \gamma_{12}ar_2) - \lambda] - \gamma_{21}br_1[(r_2 - 2br_2 - \gamma_{12}ar_2) - \lambda] \\ & \quad - \lambda[(r_2 - 2br_2 - \gamma_{12}ar_2) - \lambda] - r_1r_2\gamma_{12}\gamma_{21}ab = 0 \\ & r_1r_2 - 2br_1r_2 - \gamma_{12}ar_1r_2 - r_1\lambda - 2ar_1r_2 + 4abr_1r_2 + 2\gamma_{12}a^2r_1r_2 + 2ar_1\lambda - \gamma_{21}br_1r_2 + 2\gamma_{21}b^2r_1r_2 \\ & \quad + \gamma_{12}\gamma_{21}abr_1r_2 + \gamma_{21}br_1\lambda - r_2\lambda + 2br_2\lambda + \gamma_{12}ar_2\lambda + \lambda^2 - r_1r_2\gamma_{12}\gamma_{21}ab = 0 \\ & \quad \lambda^2 + [2ar_1 + 2br_2 + \gamma_{12}ar_2 + \gamma_{21}br_1 - r_1 - r_2]\lambda \\ & \quad + r_1r_2 + 4abr_1r_2 + 2\gamma_{12}a^2r_1r_2 + 2\gamma_{21}b^2r_1r_2 + \gamma_{12}\gamma_{21}abr_1r_2 \\ & - 2ar_1r_2 - 2br_1r_2 - \gamma_{12}ar_1r_2 - \gamma_{21}br_1r_2 - r_1r_2\gamma_{12}\gamma_{21}ab = 0 \end{aligned} \tag{14}$$

The characteristic equation of the matrix G can be represented thus,

$$H(\lambda) = \lambda^2 + u\lambda + v = 0 \tag{15}$$

where

$$u = b_1 + d_1 \text{ and } v = b_2 + d_2$$

From (14) we have that

$$\begin{aligned} b_1 &= 2ar_1 + 2br_2 + \gamma_{12}ar_2 + \gamma_{21}br_1 \\ d_1 &= -r_1 - r_2, \\ b_2 &= r_1r_2 + 4abr_1r_2 + 2\gamma_{12}a^2r_1r_2 + 2\gamma_{21}b^2r_1r_2 + \gamma_{12}\gamma_{21}abr_1r_2 \\ d_2 &= -2ar_1r_2 - 2br_1r_2 - \gamma_{12}ar_1r_2 - \gamma_{21}br_1r_2 - r_1r_2\gamma_{12}\gamma_{21}ab \end{aligned}$$

By the Routh-Hurwitz criterion, it follows that all roots of the characteristic equation (15) has negative real part if and only if the coefficients of $H(\lambda)$ be strictly positive, that is $u > 0$ ($b_1 > d_1$) and $v > 0$ ($b_2 > d_2$). Therefore the steady state $(x_1^*, x_2^*) = \frac{1}{1-\gamma_{12}\gamma_{21}}(1 - \gamma_{21}, 1 - \gamma_{12})$ is asymptotically stable if u and v are both positive.

4 Analysis of Result

The numerical simulation of the stability analysis of competition between companies based on predator prey model were conducted using Matlab 9.6.0.1072779, 2019 version to confirm the theoretical predictions discuss in section three. Thus, we showed the qualitative behavior of the two companies when both are either affected or not by competition, and we numerically solve equations (1) and (2).

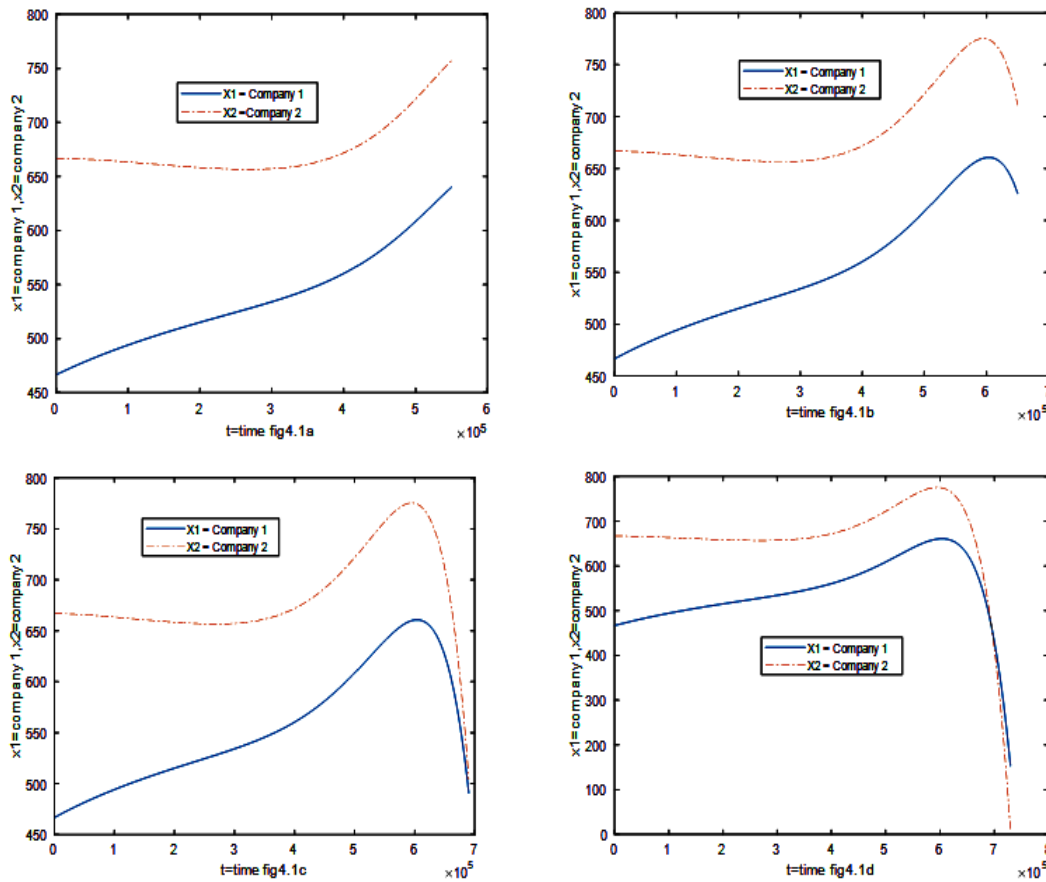


Fig. 1a – d. The behavior of x_1 and x_2 with respect to time as both companies collapse

Our observation from Fig. (1a –1d) shows that both companies gained competitive advantage in their businesses by using their available resources very effectively and this encouraged their initial growth simultaneously. But as time passes, both companies started witnessing the effect of competition which is characterized by attack and response to attack from each other. Since the rate of attack and response to attack affected them negatively and subsequently prevent them from sustaining their competitive advantage, both companies finally collapsed.

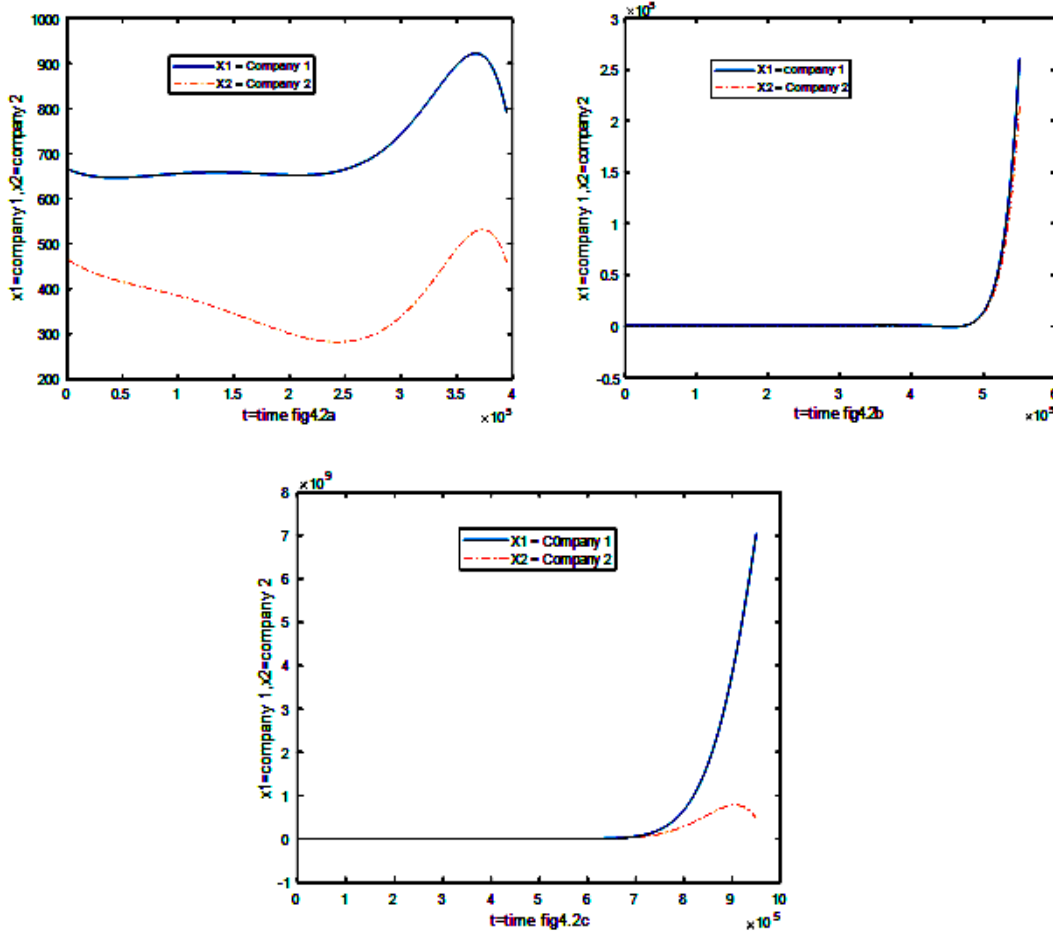


Fig. 2a – c. The behavior of x_1 and x_2 with respect to time when $\gamma_{12} > 1$ or $\gamma_{12} < 1$ and $\gamma_{21} > 1$ or $\gamma_{21} < 1$

According to the graph in Fig. 2a, we shall observe that both companies initially were affected by each other individual attack and response to attack (competition) as indicated by the oscillation. As time passes, they were able to understand how well to respond to each other’s attack and were successful in their businesses thereby growing simultaneously as were indicated in Fig. 2b. Finally, as the impact of the first company’s ($x_1 = Company 1$) competition effect (competitive advantage) on the second company ($x_2 = Company 2$) (i.e., the per capita reduction in carrying capacity of the second company caused by competition with the first company) is very high, the first company continues to grow while the second company shrinks gradually and eventually becomes extinct. This is therefore, shown in Fig. 2c that the growth of company 1 reaches equilibrium as the growth of company 2 approaches zero.

In coexistence of companies, it will be understood that if the companies making use of a resources are found to be differentiated (e.g., by size or behavior), then the resources must be divided correspondingly. And when competitors do divide a resource, the division is a permitting cause of coexistence. As a matter of fact, we can observe from Fig. 3a –3c that the coexistence of these competing companies may have resulted in some division of their resources correspondingly which may have permitted both to continuously grow simultaneously.

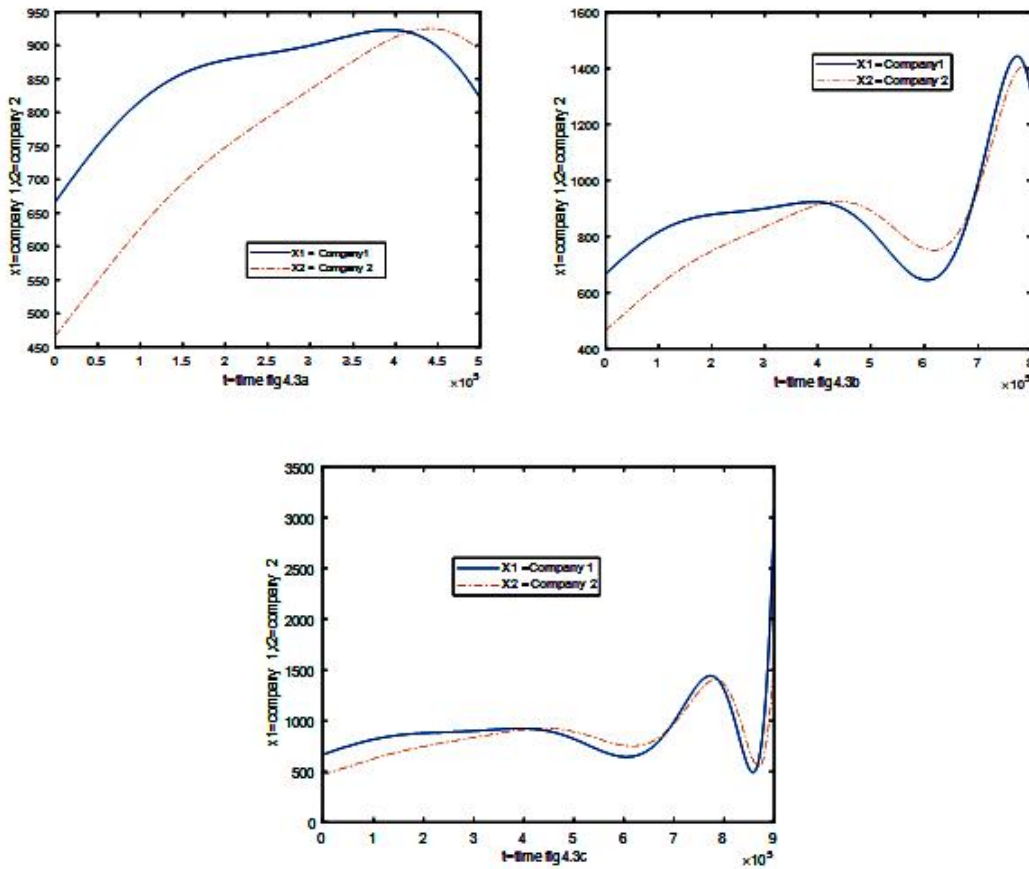


Fig. 3a – c. The behavior of x_1 and x_2 with respect to time as both companies coexist

5 Conclusion

The most interesting in the predator and prey relationship as a concept, is based on the desire to control resources and demonstrating the process on competitors. To formalize such event, we considered the predatory competitions which involves making of unprovoked attack and response to the attack and we choose the predator prey model as the basis. This model gives the possibility to consider interesting cases of company growth change, depending on resource division.

The most important result of our model is demonstration of existence of parameter values in which both companies can either coexist or one of them suffers by collapsing. The equilibrium points with parameters $b_1 > d_1$ and $b_2 > d_2$ shows that both companies can grow simultaneously (coexist), while

$\gamma_{12} > 1$ or $\gamma_{21} > 1$ implies that even the slightest change in their competitive advantage can lead to adverse situation, such as the complete disappearance of one of the companies or both.

In the case of a stable operating environment, a company can be successful with a hierarchical organizational structure which functions predictably and efficiently. In turn, in a turbulent situation, an organization should break rigid rules and the barriers between different departments and levels of hierarchy, and aim towards unprejudiced interaction [13].

Large Corporation can easily transform into bureaucratic mammoth which can be toppled during an extensive technological or market breakthrough due to their inability to adapt to the changed situation. Finally, companies can be able to save their skin and invent new products or services, or in some cases even focus their business operations on completely new fields as old opportunities become obsolete.

Competing Interests

Authors have declared that no competing interests exist.

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Appendix

From Equations (11) and (12) we have

$$x_2 = \frac{k_1 - x_1}{\gamma_{21}} \tag{11}$$

$$x_1 = \frac{k_2 - x_2}{\gamma_{12}} \tag{12}$$

$$x_1 = k_1 - \gamma_{21}x_2 \tag{11a}$$

Substituting $x_2 = k_2 - \gamma_{12}x_1$ in system (11a) we have

$$x_1 = k_1 - \gamma_{21}(k_2 - \gamma_{12}x_1)$$

$$x_1 = k_1 - \gamma_{21}k_2 + \gamma_{12}\gamma_{21}x_1$$

$$x_1(1 - \gamma_{12}\gamma_{21}) = k_1 - \gamma_{21}k_2$$

$$x_1 = \frac{k_1 - \gamma_{21}k_2}{1 - \gamma_{12}\gamma_{21}} \tag{11b}$$

$$x_2 = k_2 - \gamma_{12}x_1 \tag{12a}$$

Substituting $x_1 = k_1 - \gamma_{21}x_2$ in system (12a) we have

$$x_2 = k_2 - \gamma_{12}(k_1 - \gamma_{21}x_2)$$

$$x_2 = k_2 - \gamma_{12}k_1 + \gamma_{12}\gamma_{21}x_2$$

$$x_2(1 - \gamma_{12}\gamma_{21}) = k_2 - \gamma_{12}k_1$$

$$x_2 = \frac{k_2 - \gamma_{12}k_1}{1 - \gamma_{12}\gamma_{21}} \tag{12b}$$

Since $k_1 = k_2 = 1$, we have that systems (11b) and (12b) respectively

$$x_1 = \frac{1 - \gamma_{21}}{1 - \gamma_{12}\gamma_{21}} = (1 - \gamma_{21}) \frac{1}{1 - \gamma_{12}\gamma_{21}} \text{ and } x_2 = \frac{1 - \gamma_{12}}{1 - \gamma_{12}\gamma_{21}} = (1 - \gamma_{12}) \frac{1}{1 - \gamma_{12}\gamma_{21}}$$

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