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Reduction between Aristotelian Modal Syllogisms Based on the Syllogism ◇I□A◇I-3

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Abstract

In order to provide a consistent explanation for Aristotelian modal syllogistic, this paper reveals the reductions between the Aristotelian modal syllogism ◇I□A◇I-3 and the other valid modal syllogisms. Specifically, on the basis of formalizing Aristotelian modal syllogisms, this paper proves the validity of ◇I□A◇I-3 by means of the truth value definition of (modal) categorical propositions. Then in line with classical propositional logic and modal logic, generalized quantifier theory and set theory, this paper deduces the other 47 valid Aristotelian modal syllogisms from the modal syllogism ◇I□A◇I-3. This study shows that the reasons why these syllogisms are reducible are: 1) any of Aristotelian quantifier can be defined by the other three Aristotelian quantifiers; 2) the Aristotelian quantifiers *some* and *no* have symmetry; 3) the possible modality ◇ and necessary modality □ can be mutually defined. This formal study of Aristotelian modal syllogistic not only conforms to the needs of formalization transformation of various information in the era of artificial intelligence, but also provides a unified mathematical research paradigm for other kinds of syllogistic.

Keywords

Aristotelian Modal Syllogisms, Validity, Reduction, Possible World, Aristotelian Quantifiers

1. Introduction

In natural language, there are various kinds of syllogisms, such as Aristotelian syllogisms (Patzig, 1969; Zhang & Li, 2016; Zhou et al., 2018), Aristotelian modal syllogisms (Łukasiewicz, 1957; Zhang & Huang, 2020), and generalized syllogisms (Zhang, 2016). Therefore, syllogistic is one of the important forms of reasoning in human thinking and natural language (Hao, 2016). This paper focuses on Aristotelian modal syllogisms. In *Organon*, Aristotle studied Aristotelian modal syllogisms, many scholars have also studied them since the Middle Ages. For example, the L-X-M calculus given by McCall (1963) is a formal system to judge whether apodeictic syllogisms is valid or invalid. Johnson (1989) tried to reconstruct modal syllogistic after finding that some previous research results were inconsistent. Smith (1995) even considered that Aristotelian modal syllogistic itself is inconsistent. Thomson (1993, 1997) also failed to provide a consistent explanation for Aristotelian modal syllogistic. Johnson (2004) and Malink (2013) provided anti-models for some invalid Aristotelian modal syllogisms. Zhang (2019, 2020) conducted a formal study of Aristotelian modal syllogisms from the perspective of modern logic. Protin (2022) proposed a new modal logic with a simple deductive system to interpret Aristotle's theory of modal syllogisms.

Although many scholars have studied Aristotelian modal syllogisms, the prevailing view is that existing studies cannot give consistent explanations for Aristotelian modal syllogistic, and cannot guarantee the consistency of their results. Malink (2006) believed that the reason why previous studies cannot give consistent explanations for Aristotelian modal syllogistic is that modern modal logic and set theory are not properly applied to the syllogistic.

In the light of the generalized quantifier theory, modern modal logic and set theory, this paper tries to provide a consistent explanation for Aristotelian modal syllogistic. Specifically, this paper proves the validity of the modal syllogism ◇I□A◇I-3 on the basis of the definitions of truth value of (modal) categorical propositions, and then derives the other 47 valid modal syllogisms from the syllogism ◇I□A◇I-3 in line with some facts and inference rules.

2. Preliminaries

Aristotelian syllogisms characterize the semantic and inferential properties of the following four Aristotelian quantifiers: *all*, *no*, *some* and *not all*, which are type <1, 1> quantifiers (Zhang, 2018). The proposition containing a type <1, 1> quantifier Q can be formalized into a tripartite structure like $Q(S, P)$ (Li, 2023). In this paper, S, M and P refer to the set of objects represented by the lexical variables of categorical propositions; p, q, r and s are propositional variables. And the symbol $=_{def}$ indicates that the left can be defined by the right.

Aristotelian syllogisms contain the following four propositions that can be formalized as follows: The proposition "all S is P" is denoted by as *all* (S, P). The proposition "all S is not P" is equivalent to "no S is P" and denoted by *no* (S, P). The proposition "some S is P" is denoted by *some* (S, P). The proposition "some S is not P" is equivalent to "not all S is P" and denoted by *not all* (S, P). The above four propositions are respectively referred to as the proposition A, E, I and O (Wei, 2023).

An Aristotelian modal syllogism can be obtained by adding a possible modality ◇ or/and necessary modality □ to an Aristotelian syllogism. The definition of the figures of Aristotelian modal syllogisms are similar to that of Aristotelian syllogisms (Zhang et al., 2022). One can interpret an Aristotelian modal syllogism such as the following example:

Major premise: Some birds are possibly swallows.

Minor premise: All birds are necessarily animals.

Conclusion: Some animals are possibly swallows.

Let S represent the set composed of all animals in the domain, M the set composed of all birds in the domain, and P the set composed of all swallows in the domain. Therefore, the major premise is denoted by $\Diamond some(M, P)$, the minor premise by $\Box all(M, S)$, and the conclusion by $\Diamond some(S, P)$. Similar to the Aristotelian syllogisms, the Aristotelian modal syllogisms can be viewed as the conjunction of two premises implies the conclusion. The conjunction symbol is denoted by "$\wedge$" and the implication symbol is denoted by "$\rightarrow$". This modal syllogism in the example can be formalized as $\Diamond some(M, P) \wedge \Box all(M, S) \rightarrow \Diamond some(S, P)$. The middle term of the syllogism is the subject of the major and minor premises, so the modal syllogism is the third figure, thus it can be abbreviated as $\Diamond$I$\Box$A$\Diamond$I-3. Other syllogisms are similar.

Definition 1 (truth value definition of Aristotelian quantifiers):

1) $all(S, P) =_{def} S \subseteq P$; 2) $some(S, P) =_{def} S \cap P \neq \varnothing$;

3) $no(S, P) =_{def} S \cap P = \varnothing$; 4) $not\ all(S, P) =_{def} S \nsubseteq P$.

Definition 2 (truth value definition of modal propositions):

1) $\Box p$ is true, if and only if p is true in any possible world ω;

2) $\Diamond p$ is true, if and only if there is at least one possible world ω in which p is true.

According to modal logic (Chagrov & Zakharyaschev, 1997) and generalized quantifier theory (Peters & Westerståhl, 2006), the following facts hold:

Fact 1 (a necessary proposition implies an assertoric proposition):

1) $\Box all(S, P) \Rightarrow all(S, P)$, abbreviated as: $\Box$A$\Rightarrow$A;

2) $\Box some(S, P) \Rightarrow some(S, P)$, abbreviated as: $\Box$I$\Rightarrow$I;

3) $\Box no(S, P) \Rightarrow no(S, P)$, abbreviated as: $\Box$E$\Rightarrow$E;

4) $\Box not\ all(S, P) \Rightarrow not\ all(S, P)$, abbreviated as: $\Box$O$\Rightarrow$O.

Fact 2 (a universal proposition implies a particular proposition):

1) $all(S, P) \Rightarrow some(S, P)$, abbreviated as: A$\Rightarrow$I;

2) $no(S, P) \Rightarrow not\ all(S, P)$, abbreviated as: E$\Rightarrow$O;

3) $\Box all(S, P) \Rightarrow \Box some(S, P)$, abbreviated as: $\Box$A$\Rightarrow\Box$I;

4) $\Box no(S, P) \Rightarrow \Box not\ all(S, P)$, abbreviated as: $\Box$E$\Rightarrow\Box$O;

5) $\Diamond all(S, P) \Rightarrow \Diamond some(S, P)$, abbreviated as: $\Diamond$A$\Rightarrow\Diamond$I;

6) $\Diamond no(S, P) \Rightarrow \Diamond not\ all(S, P)$, abbreviated as: $\Diamond$E$\Rightarrow\Diamond$O.

Fact 3 (symmetry of *some* and *no*):

1) $some(S, P) \Leftrightarrow some(P, S)$; 2) $\Box some(S, P) \Leftrightarrow \Box some(P, S)$;

3) $\Diamond some(S, P) \Leftrightarrow \Diamond some(P, S)$; 4) $no(S, P) \Leftrightarrow no(P, S)$;

5) $\Box no(S, P) \Leftrightarrow \Box no(P, S)$; 6) $\Diamond no(S, P) \Leftrightarrow \Diamond no(P, S)$.

In the following, D stands for the domain of lexical variables, Q for any of the four Aristotelian quantifiers (that is, *all*, *some*, *no* and *not all*), $\neg Q$ and $Q\neg$ for the outer and inner negation of the quantifier Q, respectively.

Definition 3 (inner negation): $Q\neg(S, P) =_{\text{def}} Q(S, D\text{–}P)$.

Definition 4 (outer negation): $\neg Q(S, P) =_{\text{def}}$ It is not that $Q(S, P)$.

The following facts hold in line with Definition 3 and Definition 4:

Fact 4 (inner negation for Aristotelian quantifiers)

1) *all* (S, P) = *no*$\neg(S, P)$; 2) *no* (S, P) = *all*$\neg(S, P)$;

3) *some* (S, P) = *not all*$\neg(S, P)$; 4) *not all* (S, P) = *some*$\neg(S, P)$.

Fact 5 (outer negation for Aristotelian quantifiers):

1) $\neg$*not all* (S, P) = *all* (S, P); 2) $\neg$*all* (S, P) = *not all* (S, P);

3) $\neg$*no* (S, P) = *some* (S, P); 4) $\neg$*some* (S, P) = *no* (S, P).

Let $Q(S, P)$ is a categorical proposition, it can be seen that $\Diamond Q(S, P) =_{\text{def}} \neg\Box\neg Q(S, P)$ and $\Box Q(S, P) =_{\text{def}} \neg\Diamond\neg Q(S, P)$ in line with modal logic. Thus the following Fact 6 can be obtained:

Fact 6: 1) $\neg\Box Q(S, P) = \Diamond\neg Q(S, P)$; 2) $\neg\Diamond Q(S, P) = \Box\neg Q(S, P)$.

Aristotelian modal syllogistic is an extension of classical propositional logic, so the following rules in propositional logic can also be applied to Aristotelian modal syllogistic.

1) Rule 1 (subsequent weakening): If $\vdash(p\wedge q\rightarrow r)$ and $\vdash(r\rightarrow s)$, then $\vdash(p\wedge q\rightarrow s)$.

2) Rule 2 (anti-syllogism): If $\vdash(p\wedge q\rightarrow r)$, then $\vdash(\neg r\wedge p\rightarrow\neg q)$ or $\vdash(\neg r\wedge q\rightarrow\neg p)$.

3. Validity of the Syllogism ◇I□A◇I-3

Before discussing the reducibility of modal syllogisms, it is necessary to prove the validity of syllogism ◇I□A◇I-3.

Theorem 1 (◇I□A◇I-3): $\Diamond$*some* $(M, P)\wedge\Box$*all* $(M, S)\rightarrow\Diamond$*some* (S, P) is valid.

Proof: ◇I□A◇I-3 is the abbreviation of the modal syllogism $\Diamond$*some* $(M, P)\wedge\Box$*all* $(M, S)\rightarrow\Diamond$*some* (S, P). Suppose that $\Diamond$*some* (M, P) and $\Box$*all* (M, S) are true, then *some* (M, P) is true in at least one possible world and *all* (M, S) is true at any possible world in terms of the clause (2) and (1) in Definition 2, respectively. Thus $M \cap P \neq \varnothing$ is true in at least one possible world and $M \subseteq S$ is true at any possible world by means of the clause (2) and (1) in Definition 1, respectively. Now it follows that $S \cap P \neq \varnothing$ is true in at least one possible world. Hence *some* (S, P) in at least one possible world according to the clause (2) in Definition 1. Thus $\Diamond$*some* (S, P) is true in line with the clause (2) in Definition 2. This proves that the syllogism $\Diamond$*some* $(M, P)\wedge\Box$*all* $(M, S)\rightarrow\Diamond$*some* (S, P) is valid, just as desired.

4. The Other 47 Modal Syllogisms Derived from ◇I□A◇I-3

The following syllogisms derived from this syllogism are valid according to Theorem 1. In the following Theorem 2, ◇I□A◇I-3⇒◇I□A◇I-4 means that the modal syllogism ◇I□A◇I-4 can be deduced from the modal syllogism ◇I□A◇I-3. The others are similar.

Theorem 2: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(2.1) ◇I□A◇I-3⇒□E◇I◇O-2

(2.2) ◇I□A◇I-3⇒□E□A□E-1

Proof: For (2.1). In line with Theorem 1, it follows that ◇I□A◇I-3 is valid, and its expansion is that ◇*some* (*M*, *P*)∧□*all* (*M*, *S*)→◇*some* (*S*, *P*). According to Rule 2, it can be seen that ¬◇*some* (*S*, *P*)∧◇*some* (*M*, *P*)→¬□*all* (*M*, *S*). In the light of the clause (1) and (2) in Fact 6, it follows that □¬*some* (*S*, *P*)∧◇*some* (*M*, *P*)→◇¬*all* (*M*, *S*). With the help of the clause (2) and (4) in Fact 5, i.e., ¬*some* (*S*, *P*) = *no* (*S*, *P*) and ¬*all* (*M*, *S*) = *not all* (*M*, *S*), one can deduce that □*no* (*S*, *P*)∧◇*some* (*M*, *P*)→◇*not all* (*M*, *S*). Therefore, □E◇I◇O-2 can be derived from ◇I□A◇I-3, just as required. (2.2) can be similarly proved on the basis of the above facts and rules.

Theorem 3: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(3.1) ◇I□A◇I-3⇒◇I□A◇I-4

(3.2) ◇I□A◇I-3⇒□A◇I◇I-3

(3.3) ◇I□A◇I-3⇒□A◇I◇I-1

(3.4) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-4

(3.5) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-1

(3.6) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-3

(3.7) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2

(3.8) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-4

(3.9) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-2

Proof: For (3.1). As pointed out earlier, ◇I□A◇I-3 is valid, which is the abbreviation of the modal syllogism ◇*some* (*M*, *P*)∧□*all* (*M*, *S*)→◇*some* (*S*, *P*). According to clause (3) in Fact 3: ◇*some* (*M*, *P*)↔◇*some* (*P*, *M*). Hence, it follows that ◇*some* (*P*, *M*)∧□*all* (*M*, *S*)→◇*some* (*S*, *P*). That is to say that ◇I□A◇I-4 can be derived from ◇I□A◇I-3, the proof of (3.1) has been completed. The remaining syllogisms can be similarly inferred from ◇I□A◇I-3.

Theorem 4: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(4.1) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□AE-2

(4.2) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-4⇒□A□EE-4

(4.3) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-2⇒□A□EE-2

(4.4) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1

Proof: For (4.1). According to (3.7) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2, it follows that □E□A□E-2 is valid, and its expansion is that □*no* (*P*, *M*)∧□*all* (*S*, *M*)→□*no* (*S*, *P*). According to clause (3) in Fact 1: □*no* (*S*, *P*)⇒*no* (*S*, *P*). Then it follows that □*no* (*P*, *M*)∧□*all* (*S*, *M*)→*no* (*S*, *P*) from □E□A□E-2 on the basis of Rule 1. In other words, the modal syllogism □E□AE-2 is valid, as required. Others can be similarly proved.

Theorem 5: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(5.1) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□O-1

(5.2) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒□E□AO-1

(5.3) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□A□O-2

(5.4) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-4⇒□A□E□O-4

(5.5) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-2⇒□A□E□O-2

(5.6) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□AE-2⇒□E□AO-2

(5.7) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-4⇒□A□EE-4⇒□A□EO-4

(5.8) ◇I□A◇I-3⇒□E□A□E-1⇒□A□E□E-2⇒□A□EE-2⇒□A□EO-2

Proof: For (5.1). In terms of (2.2) ◇I□A◇I-3⇒□E□A□E-1, it can be seen that □E□A□E-1 is valid. With the help of clause (2) in Fact 2, it follows that E⇒O, so □E□A□O-1 is valid. In other words, □E□A□O-1 can be derived from ◇I□A◇I-3, as required. All of the other syllogisms can be similarly deduced from ◇I□A◇I-3 by means of the above theorems, facts and rules.

Theorem 6: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(6.1) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-1⇒□A◇I◇I-1

(6.2) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-3⇒□A◇I◇I-3

(6.3) ◇I□A◇I-3⇒◇O□A◇O-3

Proof: For (6.1). With the help of (3.5) ◇I□A◇I-3⇒□E◇I◇O-2⇒□E◇I◇O-1, it follows that □E◇I◇O-1 is valid, and its expansion is that □*no* (*M*, *P*)∧◇*some* (*S*, *M*)→◇*not all* (*S*, *P*). According to the clause (2) and (4) in Fact 4, *no* (*M*, *P*) = *all*¬(*M*, *P*), and *not all* (*S*, *P*) = *some*¬(*S*, *P*), it follows that □*all*¬(*M*, *P*)∧◇*some* (*S*, *M*)→◇*some*¬(*S*, *P*) from ◇I□A◇I-3. According to Definition 3, *all*¬(*M*, *P*) = *all* (*M*, *D*–*P*), *some*¬(*S*, *P*) = *some* (*S*, *D*–*P*). Therefore, one can derive that □*all* (*M*, *D*–*P*)∧◇*some* (*S*, *M*)→◇*some* (*S*, *D*–*P*). In other words, the syllogism □A◇I◇I-1 can be derived from ◇I□A◇I-3, just as desired. Similarly, (6.2) and (6.3) can be proved.

Theorem 7: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(7.1) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□A□O-2⇒□A◇A◇I-3

(7.2) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□AE-2⇒□E□AO-2⇒□AA◇I-3

(7.3) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒□EI◇O-2

(7.4) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3

(7.5) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A◇O◇O-2

(7.6) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1

Proof: For (7.1). According to (5.3) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□A□O-2, it follows that □E□A□O-2 is valid, and its expansion is that □*no* (*P*, *M*)∧□*all* (*S*, *M*)→□*not all* (*S*, *P*). According to Rule 2, it can be seen that ¬□*not all* (*S*, *P*)∧□*all* (*S*, *M*)→¬□*no* (*P*, *M*). In the light of the clause (1) and (2) in Fact 6, it follows that ◇¬*not all* (*S*, *P*)∧□*all* (*S*, *M*)→◇¬*no* (*P*, *M*). With the help of the clause (1) and (3) in Fact 5, i.e., ¬*not all* (*S*, *P*) = *all* (*S*, *P*) and ¬*no* (*P*, *M*) = *some* (*P*, *M*), one can deduce that □*all* (*S*, *M*)∧◇*all* (*S*, *P*)→◇*some* (*P*, *M*). Therefore, □A◇A◇I-3 can be derived from ◇I□A◇I-3. Others can be similarly proved.

Theorem 8: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(8.1) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□A□O-2⇒□A◇A◇I-3⇒◇A□A◇I-3

(8.2) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3⇒I□A◇I-4

(8.3) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3⇒I□A◇I-4⇒I□A◇I-4

(8.4) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3⇒I□A◇I-4⇒I□A◇I-4⇒□AI◇I-1

(8.5) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3⇒□AI◇I-3

(8.6) ◇I□A◇I-3⇒□E□A□E-1⇒□E□AE-1⇒I□A◇I-3⇒O□A◇O-3

Proof: For (8.1). In terms of (7.1) ◇I□A◇I-3⇒□E□A□E-1⇒□E□A□E-2⇒□E□A□O-2⇒□A◇A◇I-3, one can obtain that □A◇A◇I-3 is valid, and its expansion is that □*all* (*M*, *P*)∧◇*all* (*M*, *S*)→◇*some* (*S*, *P*). In the light of the clause (3) in Fact 3, it follows that ◇*some* (*S*, *P*)↔◇*some* (*P*, *S*). Therefore, it is easily seen that ◇*all* (*M*, *S*)∧□*all* (*M*, *P*)→◇*some* (*P*, *S*). That is, ◇A□A◇I-3 can be derived from ◇I□A◇I-3, the proof of (8.1) has been completed. The others can be similarly followed from ◇I□A◇I-3.

Theorem 9: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(9.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□AA-1

(9.2)◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1⇒□A□A□I-4⇒□A□AI-4

Proof: For (9.1). According to (7.6) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1, it follows that □A□A□A-1 is valid. According to clause (1) in Fact 1, it is easily seen that □A⇒A, so □A□AA-1 is valid. In other words, it can be derived from ◇I□A◇I-3, just as desired. (9.2) can be similarly proved.

Theorem 10: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(10.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1

(10.2) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□AA-1⇒□A□AI-1

Proof: For (10.1). With the help of (7.6) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1, it can be seen that □A□A□A-1 is valid. With the help of clause (1) in Fact 2, it follows that A⇒I, so □A□A□I-1 is valid. That is to say that □A□A□I-1 can be derived from ◇I□A◇I-3, as required. On the basis of the above theorems, facts and rules, (10.2) can be similarly deduced from ◇I□A◇I-3.

Theorem 11: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(11.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1⇒□A□A□I-4

(11.2) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1⇒◇E□A◇O-3⇒◇E□A◇O-4

(11.3) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□AA-1⇒□A□AI-1⇒E□A◇O-3⇒E□A◇O-4

Proof: For (11.1). According to (10.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1, □A□A□I-1 is valid, which is the abbreviation of the modal syllogism □*all* (*M*, *P*) ∧□*all* (*S*, *M*)→◇*some* (*S*, *P*). In the light of the clause (2) in Fact 3, it follows that □*some* (*S*, *P*)↔□*some* (*P*, *S*). Therefore, it is easily seen that □*all* (*S*, *M*)∧□*all* (*M*, *P*)→□*some* (*P*, *S*). That is, □A□A□I-4 can be derived from ◇I□A◇I-3, the proof of (12.1) has been completed. The others can be similarly followed from ◇I□A◇I-3.

Theorem 12: The following valid modal syllogisms can be deduced from ◇I□A◇I-3:

(12.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1⇒◇E□A◇O-3

(12.2) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□AA-1⇒□A□AI-1⇒E□A◇O-3

Proof: For (7.1). In the light of (10.1) ◇I□A◇I-3⇒◇O□A◇O-3⇒□A□A□A-1⇒□A□A□I-1, it follows that □A□A□I-1 is valid, and its expansion is that □*all* (*M*, *P*)∧□*all* (*S*, *M*)→□*some* (*S*, *P*). According to Rule 2, it can be seen that ¬□*some* (*S*, *P*)∧□*all* (*S*, *M*)→¬□*all* (*M*, *P*). In line with the clause (1) and (2) in Fact 6, it follows that ◇¬*some* (*S*, *P*)∧□*all* (*S*, *M*)→◇¬*all* (*M*, *P*). With the help of the clause (4) and (2) in Fact 5, i.e., ¬*some* (*S*, *P*) = *no* (*S*, *P*) and ¬*all* (*M*, *P*) = *not all* (*M*, *P*), one can deduce that ◇*no* (*S*, *P*)∧□*all* (*S*, *M*)→◇*not all* (*M*, *P*). Therefore, ◇E□A◇O-3 can be derived from ◇I□A◇I-3. (12.2) can be similarly proved on the basis of the above theorems, facts and rules.

So far, this paper has deduced the validity of 47 modal syllogisms from that of the syllogism ◇I□A◇I-3 according to reduction operations between different syllogisms.

5. Conclusion

In order to provide a consistent explanation for Aristotelian modal syllogistic, this paper reveals the reductions between the modal syllogism ◇I□A◇I-3 and the other valid modal syllogisms on the basis of generalized quantifier theory, modern modal logic and set theory. Specifically, this paper proves the validity of the modal syllogism ◇I□A◇I-3 in the light of the definitions of truth value of modal categorical propositions, and then derives the other 47 valid modal syllogisms from the syllogism in line with some facts and inference rules. The reason why these syllogisms are reducible is that: 1) any of Aristotelian quantifier can be defined by the other three Aristotelian quantifiers; 2) the Aristotelian quantifiers *some* and *no* have symmetry; 3) the possible modality ◇ and necessary modality □ can be mutually defined.

From the perspective of mathematical structuralism, holism and system optimization, this paper gives a formal study of Aristotelian modal syllogistic, which not only conforms to the needs of formalization transformation of various information in the era of artificial intelligence, but also provides a unified mathematical research paradigm for other kinds of syllogisms, such as generalized syllogisms, relational syllogisms, generalized modal syllogisms, syllogisms with verbs, syllogisms with adjectives, and syllogisms with Boolean operations, and so on. As for future research, we can consider how to use the research methods of this paper to formally study other kinds of syllogisms, such as generalized syllogisms and generalized modal syllogisms.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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