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The Application of Modified F-expansion Method Solving the Maccari's System

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Abstract

The present article investigates the use of a modified F-expansion method in finding the exact traveling wave solution of two-component nonlinear partial differential equations (NLPDEs). More specifically, this method is used to construct new solutions to the nonlinear Maccari's system (1+2)-dimensional. The solutions obtained can exactly generate soliton solutions, triangular periodic wave solutions, exponential and rational solutions unther some certain condition. In addition, some fig-uses of partial solutions for direct-viewing analysis are suggested.

Keywords: Modified F-expansion method; exact solution; Maccari's system (2+1)-dimensional.

2010 Mathematics Subject Classification: 35Q53; 35Q80; 35Q55; 35G25.

1 Introduction

The discovery of the soliton, its remarkable properties and the incredible richness of structure are all included in its mathematical description. The story begins with the observation by John Scott Russell of "the great wave of translation". It was not till the 1870's that Russell's work was finally vindicated and its scientific importance was appreciated by some eminent scholars. Independently, Boussinesq [1] (1872) and Rayleigh (1876) found the hyperbolic secant squared solution for the free surface. Boussinesq's 1872 paper, in fact, did a lot more and introduced many of the ideas nowadays used by modem analysts. In particular, he

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found the conserved density of the third conservation law, a quantity he called the moment of instability. He derived his solution from the approximation to the water wave equations that now bear his name. In this approximation, the motion can be bidirectional 1 but the basic idea of the balance between nonlinearity and dispersion is present. It was left to Korteweg and deVries in 1895, who apparently did not know the work of Boussinesq and Rayleigh and who were still trying to answer the objections of Airy and Stokes, to write down the unidirectional equation which now bears their names. (It would appear to have been the thesis project of deVries.) In this first stage of discovery, the primary thrust was to establish the existence and resilience of the wave. The discovery of its universal nature and its additional properties was to await a new day and an unexpected result from another experiment designed to answer a totally different question.

The appearance of solitary wave solutions in nature is quite is common. Refereces can be made to Bell-shaped sech-solutions and kink-shaped tanh-solutions model wave phenomena in fluids, plasmas, elastic media, electrical circuits, optical fibers, chemical reactions, bio-genetics, etc. The travelling wave solutions of the Korteweg-de Vries (KdV) and Boussionesq equations, which describe water waves, are famous examples as well. For a more detailed and technical account of the solitary wave, see [2].

In recent years, other methods have developed, such as the Backlund transformation method[3], Darboux transformation [4], tanh method [5,6] extended tanh function method [7], Exp-function method [8], the generalize hyperbolic function [9], the first integral method [10], the exp(-Φ(ξ))-expansion method [11], enhanced (G'/G)-expansion method [12-15], modified simple equation method [16], and the *F*-expansion method [17,18]. All the above-mentioned approaches are based on the assumption that the solutions can be uniformly expressed in terms of some special ansatz. Therefore, the original partial differential equations (PDE’s) can be transformed into a set of algebraic equations through balancing the same order of the ansatz, which yields the explicit expressions of the waves. The difference between these methods is attributed to the different ansatz introduced. For example, in the tanh-coth method, the ansatz can be written in as combinations of tanh and coth functions, whild in the Jacobi elliptic function expansion method, the ansatz can be expressed in the form of Jacobi elliptic functions. From our point of view, all these methods have some nerits and demerits with respect to the problem considered and there is no unified method that can be used to deal with all types of NLPDEs. That is why anytime that an improvement is made in a particular method to allow it to recover some new solutions to the NLPDEs, it is always welcomed. The purpose of this paper is to apply a modified F-expansion method to coupled familied of NLPDEs.

The aim of this paper is organized as follows: In Section 2,at first, we briefly introduse the steps involved in the modified F-expansion method, In Section 3, by using the results obtained in Section 2, attempts are made to apply the method to solve the Maccari’s system (2+1)-dimensional.

2 Description of Method

Consider a nonlinear partial differential equation with independent variables $x=(t,x_1,x_2,...,x_m)$ and dependent variables u, in the form:

$$F(u,u_t,u_{x_1},u_{x_2},...,u_{x_m},u_{x_1},u_{x_2},...,u_{tx_m},u_{tt},u_{x_1x_1},u_{x_2x_2},...,u_{x_mxm},...)=0, \tag{1}$$

where $u=u(x,y,t)$ is the solution of nonlinear partial differential Eq.(1). Furthermore, the transformations which are used are as follows:

$$u(x_1,x_2, \ , x_m,t)=U(\xi), \qquad \zeta = k_1(x_1+k_2x_2+...+k_mx_m - \lambda t). \tag{2}$$

Where λ and k_i are constants. Using the chain rule, it can be found that

$$\frac{\partial}{\partial t}(.)=-k_1\lambda\frac{\partial}{\partial\xi}(.),\quad \frac{\partial}{\partial x_1}(.)=k_1\frac{\partial}{\partial\xi}(.),\quad \frac{\partial}{\partial x_2}(.)=k_1k_2\frac{\partial}{\partial\xi}(.),\quad \frac{\partial^2}{\partial x_1^2}(.)=k_1^2\frac{\partial^2}{\partial\xi^2}(.),\ \ldots \tag{3}$$

At present, Eq. (3) is used to show transfering the nonlinear partial differential equation Eq. (1) to nonlinear ordinary differential equation:

$$H\left(U(\xi),U_\xi(\xi),U_{\xi\xi}(\xi),\ldots\right)=0. \tag{4}$$

According to the modified F-expansion method, it is assumed that the solution can be expressed in the form:

$$U(\xi)=a_0+\sum_{i=1}^{N}a_iF^i(\xi)+\sum_{i=1}^{N}b_iF^{-i}(\xi) \tag{5}$$

where a_0, a_i and b_i are constants to be determined. $F(\xi)$ satisfies Riccati equation:

$$F'(\xi)=A+BF(\xi)+CF^2(\xi) \tag{6}$$

where A, B and C are constants to be determined. The prime $(')$ denotes $d/d\xi$. Integer N can be determined by considering the homogeneous balance between the governing nonlinear term (s) and highest order derivatives of $U(\xi)$ in Eq.(4). Given different values of A, B and C, the different Riccati function solution $F(\xi)$ can be obtained from Eq. (6) (see Table 1). To determine $U(\xi)$ explicitly, we take the following steps:

Step I. Substituting (5) along with (6) into Eq. (4) and collect coefficients of $F^i(\xi)$ to zero yields a system of algebraic equations for $a_i\,(i=N,\ldots,1,0)$, $b_i\,(i=1,\ldots,N)$, $k_i\,(i=1,\ldots,m)$ and λ.

Step II. Solve the system of algebraic equations, probably with the aid of Mathematica or Maple. $a_i\,(i=N,\ldots,1,0)$ and $b_i\,(i=1,\ldots,N)$ can be expressed by A, B and C (or the coefficients of ODE(4). Substituting these results into (5), we can obtain the general form of travelling wave solutions to Eq.(4).

Step III. Selecting A, B, C and $F(\xi)$ from Table 1 and substituting them along with $a_i\,(i=N,\ldots,1,0)$ and $b_i\,(i=1,\ldots,N)$ into Eq.(5), a series of soliton-like solutions, trigonmetric function solutions and rational solutions to Eq.(4) can be obtained.

The modified F-expansion method is more effective in obtaining the soliton-like solution, trigonometric function solutions, exponential solutions and rational solutions of the nonlinear partial deferential equations. This method will yield more rich types solutions of the nonlinear partial deferential equations. It shows that the modified F-expansion method is more powerful in constructing exact solutions of NLPDEs.

Relations between values of A, B, C and corresponding $F(\xi)$ in Eq.(6) are listed in (Table 1) [17].

Table 1. Relations between values of A, B, C and corresponding $F(\xi)$ in Eq. (6)

Values of A, B, C	$F(\xi)$
$A=0$, $B=1$, $C=-1$	$\frac{1}{2}+\frac{1}{2}tanh(\frac{1}{2}\xi)$
$A=0$, $B=-1$, $C=1$	$\frac{1}{2}-\frac{1}{2}coth(\frac{1}{2}\xi)$
$A=\frac{1}{2}$, $B=0$, $C=\frac{-1}{2}$	$coth(\xi)\pm \text{csch}(\xi), \tanh(\xi)\pm \text{isech}(\xi)$
$A=1$, $B=0$, $C=-1$	$\tanh(\xi), \coth(\xi)$
$A=\frac{1}{2}$, $B=0$, $C=\frac{1}{2}$	$sec(\xi)+tan(\xi), \csc(\xi)-\cot(\xi)$
$A=\frac{1}{2}$, $B=0$, $C=\frac{1}{2}$	$sec(\xi)-tan(\xi), \csc(\xi)+\cot(\xi)$
$A=1(-1)$, $B=0$, $C=1(-1)$	$\tan(\xi), \cot(\xi)$
$A=0$, $B=1$, $C\neq 0$	$\frac{-1}{C\xi+\eta}$ (η is an arbitrary constant)
A is arbitrary constant, $B=0$, $C=0$	$A\xi$
A is arbitrary constant, $B\neq 0$, $C=0$	$\frac{exp(B\xi)-A}{B}$

3 Exact Solutions of Maccari's System

In this section, we discuss the Maccari's System (2+1)-dimensional system, written in the form of the following equations:

$$\begin{cases} iu_t+u_{xx}+uv=0, \\ u_t+u_y+\left(|u|^2\right)_x=0. \end{cases} \tag{7}$$

The celebrated (2+1)-dimensional Maccari's system has been found in some studies conducted by Maccari. Maccari derived this system from the Kadomtsev-Petvishvili equation by using the asymptotically exact reduction method based Fourier expansion and spatiotemporal rescaling [19]. It is necessary to state that equation (7) plays an important role in nonlinear physics. That is to say, this system is often presented to describe the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, etc.

By using the transformation:

$$u=u(x,y,t)=exp(i\theta)U(\xi),\ v=v(x,y,t)=V(\xi),\ \xi=k(x+ly-\lambda t),\ \theta=\alpha x+\beta y+\gamma t. \tag{8}$$

where k,l,λ,α,β and γ are arbitrary constants, substituting Eq.(8) with Eq.(7), there will be a change into:

$$\begin{cases} i\left(i\gamma U(\xi)-k\lambda U_\xi(\xi)\right)e^{i\theta}+\left(2i\alpha kU_\xi(\xi)+k^2U_{\xi\xi}(\xi)-\alpha^2U(\xi)\right)e^{i\theta}+e^{i\theta}U(\xi)V(\xi)=0, \\ \gamma V_\xi(\xi)+klV_\xi(\xi)+k\left(U^2(\xi)\right)_\xi=0. \end{cases} \tag{9}$$

where by integrating the second equation of the Eq.(9), respect to ξ, it can be obtained that:

$$V(\xi) = -\frac{k}{\gamma + kl} U^2(\xi) + \frac{d_1}{\gamma + kl}. \tag{10}$$

where d_1 is an integral constant. Substituting Eq.(10) with the first equation of the Eq.(9) results in:

$$i(2\alpha k - k\lambda)U_\xi(\xi) + (\frac{d_1}{\gamma + kl} - \gamma - \alpha^2)U(\xi) - \frac{k}{\gamma + kl}\mathrm{U}^3(\xi) + k^2 \mathrm{U}_{\xi\xi}(\xi) = 0, \tag{11}$$

if we let the $\lambda = 2\alpha$, it can be found that:

$$(\frac{d_1}{\gamma + kl} - (\gamma + \alpha^2))U(\xi) - \frac{k}{\gamma + kl}\mathrm{U}^3(\xi) + k^2 \mathrm{U}_{\xi\xi}(\xi) = 0. \tag{12}$$

Considering the homogeneous balance between U^3 and $\mathrm{U}_{\xi\xi}$ in (12), we suppose that the solution to ordinary differential equation (12) can be expressed by:

$$U(\xi) = a_0 + a_1 F(\xi) + b_1 F^{-1}(\xi) \tag{13}$$

where a_0, a_1 *and* b_1 are constants to be determined. Substituting (13) with Eq. (12), and using (6), the left-hand side of Eq. (13) can be converted into a finite series in $F^j(\xi)\ (j = -3, \ , 3)$. Equating each coefficient of $F^j(\xi)$ to zero yields a system of algebraic equations for a_0, a_1, b_1 and d_1:

$$\begin{aligned}
&F^3 : -\frac{k}{\gamma + kl} a_1^3 + 2k^2 a_1 C^2, \\
&F^2 : -3\frac{k}{\gamma + kl} a_0 a_1^2 + 3k^2 a_1 BC, \\
&F : a_1(\frac{d_1}{\gamma + kl} - \left(\gamma + \alpha^2\right)) - 3\left(a_1^2 b_1 + a_0^2 a_1\right)\frac{k}{\gamma + kl} + k^2 a_1\left(B^2 + 2AC\right), \\
&F^0 : a_0(\frac{d_1}{\gamma + kl} - (\gamma + \alpha^2)) - \frac{k}{\gamma + kl}\left(a_1^3 + 3a_0 a_1 b_1\right) + k^2\left(a_1 AB + CBb_1\right), \\
&F^{-1} : b_1(\frac{d_1}{\gamma + kl} - (\gamma + \alpha^2)) - 3\frac{k}{\gamma + kl}\left(a_1 b_1^2 + a_0^2 b_1\right) + k^2\left(B^2 b_1 + 2b_1 AC\right), \\
&F^2 : -3\frac{k}{\gamma + kl} a_0 b_1^2 + 3k^2 ABb_1, \\
&F^{-3} : -\frac{kb_1^3}{\gamma + kl} + 2k^2 A^2 b_1.
\end{aligned} \tag{14}$$

Solving the algebraic equations (14) using Maple, the following solutions will be obtained:

Case I:

$$a_0 = B\sqrt{\frac{k\gamma + k^2 l}{2}}, \qquad a_1 = 0, \qquad b_1 = A\sqrt{\frac{k\gamma + k^2 l}{2}}$$

$$d_1 = \gamma^2 + (k^2\gamma + k^3 l)(\frac{1}{2}B^2 - 2AC) + \alpha^2\gamma + kl\left(\gamma + \alpha^2\right). \tag{15}$$

Case II:

$$a_0 = \frac{kB(\gamma + kl)}{\sqrt{2(k\gamma + k^2 l)}}, \qquad a_1 = C\sqrt{2(k\gamma + k^2 l)}, \qquad b_1 = 0$$

$$d_1 = \gamma^2 + (k^2\gamma + k^3 l)(\frac{1}{2}B^2 - 2AC) + \alpha^2\gamma + kl(\gamma + \alpha^2). \tag{16}$$

Substituting (15) and (16) with Eq.(13), from Table 1, we may obtain many soliton-like solutions, trigonometric function solutions, exponential solutions and rational solutions to Eq.(7) (where we left the same type solutions out):

3.1 The Soliton-Like Solutions to Maccari's System

(1) When $A = 0,\ B = 1,\ C = -1,$ from Table 1, $F(\xi) = \frac{1}{2} + \frac{1}{2}tanh\left(\frac{1}{2}\xi\right)$. By (16), the exact solution to equation (7) is given by:

$$u_1(x,y,t) = \left(\sqrt{\frac{k\gamma + k^2 l}{2}} - \sqrt{2(k\gamma + k^2 l)}(\frac{1}{2} + \frac{1}{2}tanh(\frac{1}{2}(k(x + ly - \lambda t)))\right)e^{i(\alpha x + \beta y + \gamma t)},$$

$$v_1(x,y,t) = -\frac{k}{\gamma + kl}u_1^2(x,y,t) + (\gamma + \alpha^2 + \frac{k^2}{2}). \tag{17}$$

(2) When $A = 0,\ B = -1,\ C = 1,$ from Table 1, $F(\xi) = \frac{1}{2} - \frac{1}{2}coth\left(\frac{1}{2}\xi\right)$. By (16), the exact solution to equation (7) is given by:

$$u_2(x,y,t) = \left(-\sqrt{\frac{k\gamma + k^2 l}{2}} + \sqrt{2(k\gamma + k^2 l)}(\frac{1}{2} - \frac{1}{2}coth(\frac{1}{2}(k(x + ly - \lambda t)))\right)e^{i(\alpha x + \beta y + \gamma t)},$$

$$v_2(x,y,t) = -\frac{k}{\gamma + kl}u_2^2(x,y,t) + (\gamma + \alpha^2 + \frac{k^2}{2}). \tag{18}$$

(3) When $A = \frac{1}{2},\ B = 0,\ C = -\frac{1}{2},$ from Table 1, $F(\xi) = coth(\xi) \pm csc\,h(\xi)\ or\ tanh(\xi) \pm i\,sec\,h(\xi).$ By (15), the exact solution to equation (7) is given by:

$$u_3(x,y,t)=\left(\frac{\sqrt{k\gamma+k^2 l}}{2\sqrt{2}(coth(k(x+ly-\lambda t))\pm csc\,h(k(x+ly-\lambda t)))}\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_3(x,y,t)=-\frac{k}{\gamma+kl}u_3^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{19}$$

$$u_4(x,y,t)=\left(\frac{\sqrt{k\gamma+k^2 l}}{2\sqrt{2}(tanh(k(x+ly-\lambda t))\pm i\,sec\,h(k(x+ly-\lambda t)))}\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_4(x,y,t)=-\frac{k}{\gamma+kl}u_4^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{20}$$

By case (16), the exact solution to equation (7) can be written as:

$$u_5(x,y,t)=\left(-\frac{\sqrt{k\gamma+k^2 l}}{2}(coth(k(x+ly-\lambda t))\pm csc\,h(k(x+ly-\lambda t)))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_5(x,y,t)=-\frac{k}{\gamma+kl}u_5^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{21}$$

$$u_6(x,y,t)=\left(-\frac{\sqrt{k\gamma+k^2 l}}{2}(tanh(k(x+ly-\lambda t))\pm i\,sec\,h(k(x+ly-\lambda t)))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_6(x,y,t)=-\frac{k}{\gamma+kl}u_6^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{22}$$

(4) When $A=1,\ B=0,\ C=-1,$ from Table 1, $F(\xi)=tanh(\xi)\,or\,coth(\Xi)$. By (15), the exact solution to equation (7) can be written as:

$$u_6(x,y,t)=\left(\sqrt{\frac{k\gamma+k^2 l}{2}}\,coth(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_7(x,y,t)=-\frac{k}{\gamma+kl}u_7^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{23}$$

$$u_8(x,y,t)=\left(\sqrt{\frac{k\gamma+k^2 l}{2}}\,tanh(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_8(x,y,t)=-\frac{k}{\gamma+kl}u_8^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{24}$$

By (16), the exact solution to equation (7) is given by:

$$u_9(x,y,t)=\left(-\sqrt{2(k+k^2l)}\,tanh(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_9(x,y,t)=-\frac{k}{\gamma+kl}u_9^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{25}$$

$$u_{10}(x,y,t)=\left(-\sqrt{2(k\gamma+k^2l)}\,coth(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{10}(x,y,t)=-\frac{k}{\gamma+kl}u_{10}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{26}$$

For direct-viewing analysis, we provide the figures of $u_5(x,y,t)$, where we choose $\alpha=\beta=\gamma=\lambda=0,\ l=1 \text{ and } k=2.$

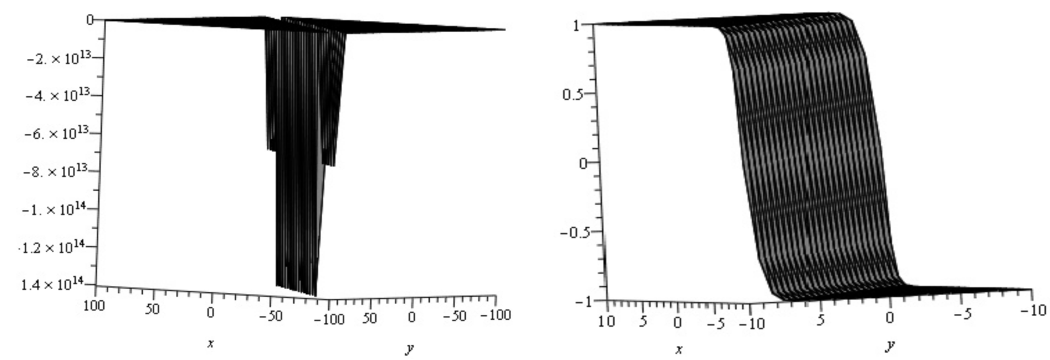

Fig. 1. Graphics of soliton-like solution u_5 are shown at "+" and "−", respectively

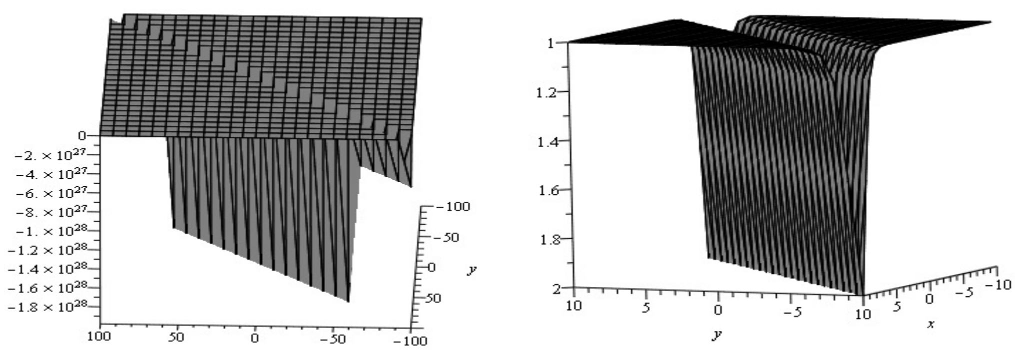

Fig. 2. Graphics of soliton-like solution u_5 are shown at "+" and "−", respectively

3.2 The Trigonometric Function Solutions to Maccari's System

(1) When $A=\frac{1}{2},\ B=0,\ C=\frac{1}{2}$, from Table 1, $F(\xi)=sec(\xi)+tan(\xi)\ or\ csc(\xi)-cot(\xi)$. By (16), the exact solution to equation (7) can be written as:

$$u_{11}(x,y,t)=\left(\frac{1}{2}\sqrt{2(k\gamma+k^2l)}(sec(k(x+ly-\lambda t))+tan(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{11}(x,y,t)=-\frac{k}{\gamma+kl}u_{11}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{27}$$

$$u_{12}(x,y,t)=\left(\frac{1}{2}\sqrt{2(k\gamma+k^2l)}(csc(k(x+ly-\lambda t))-cot(k(x+ly-\lambda t))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{12}(x,y,t)=-\frac{k}{\gamma+kl}u_{12}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{28}$$

By (15), the exact solution to equation (7) can be written as:

$$u_{13}(x,y,t)=\frac{\sqrt{k\gamma+k^2l}}{2\sqrt{2}(sec(k(x+ly-\lambda t))+tan(k(x+ly-\lambda t)))}e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{13}(x,y,t)=-\frac{k}{\gamma+kl}u_{13}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{29}$$

$$u_{14}(x,y,t)=\frac{\sqrt{k\gamma+k^2l}}{2\sqrt{2}(csc(k(x+ly-\lambda t))-cot(k(x+ly-\lambda t)))}e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{14}(x,y,t)=-\frac{k}{\gamma+kl}u_{14}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{30}$$

(2) When $A=-\frac{1}{2},\ B=0,\ C=-\frac{1}{2},$ from Table 1, $F(\xi)=sec(\xi)-tan(\xi)\ \ or\ \ csc(\xi)+cot(\xi)$. By (15), the exact solution to equation (7) will be shown as follows:

$$u_{15}(x,y,t)=-\frac{\sqrt{k\gamma+k^2l}}{2\sqrt{2}(sec(k(x+ly-\lambda t))-tan(k(x+ly-\lambda t)))}e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{15}(x,y,t)=-\frac{k}{\gamma+kl}u_{15}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{31}$$

$$u_{16}(x,y,t)=-\frac{\sqrt{k\gamma+k^2l}}{2\sqrt{2}(csc(k(x+ly-\lambda t))+cot(k(x+ly-\lambda t)))}e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{16}(x,y,t)=-\frac{k}{\gamma+kl}u_{16}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{32}$$

By (16), the exact solution to equation (7) can be written as:

$$u_{17}(x,y,t)=\left(-\frac{1}{2}\sqrt{2(k\gamma+k^2l)}(sec(k(x+ly-\lambda t))-tan(k(x+ly-\lambda t)))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{17}(x,y,t)=-\frac{k}{\gamma+kl}u_{17}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \quad (33)$$

$$u_{18}(x,y,t)=\left(-\frac{1}{2}\sqrt{2(k\gamma+k^2l)}(csc(k(x+ly-\lambda t))+cot(k(x+ly-\lambda t)))\right)e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{18}(x,y,t)=-\frac{k}{\gamma+kl}u_{18}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \quad (34)$$

(3) When $A=1,\ B=0,\ C=1,$ from Table 1, $F(\xi)=tan(\xi)$. By (16), the exact solution to equation (7) is found to be:

$$u_{19}(x,y,t)=\sqrt{2(k\gamma+k^2l)}\,tan(k(x+ly-\lambda t))e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{19}(x,y,t)=-\frac{k}{\gamma+kl}u_{19}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \quad (35)$$

For direct-viewing analysis, we provide the figures of $u_{11}(x,t)$, $v_{11}(x,t)$, $u_{12}(x,t)$, and $v_{12}(x,t)$, where we choose $l=\beta=0,\ \alpha=\gamma=-1,\ k=-2\ and\ \lambda=1.$

By (15), the exact solution to equation (7) is obtained as:

$$u_{20}(x,y,t)=\sqrt{\frac{1}{2}(k\gamma+k^2l)}\,cot(k(x+ly-\lambda t))e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{20}(x,y,t)=-\frac{k}{\gamma+kl}u_{20}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \quad (36)$$

where $\alpha,\beta,\gamma,k,l\ and\ \lambda$ are real constants in section 2.1 and 2.2.

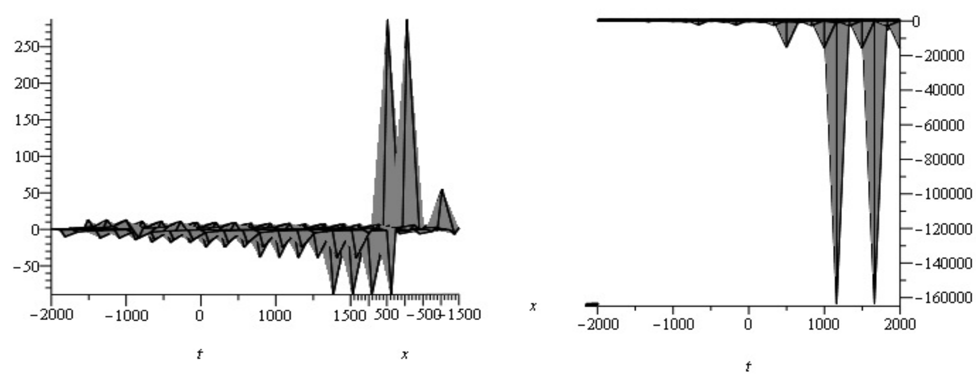

Fig. 3. (a) Graphics of the periodic solution u_{11} (b) Graphics of the periodic solution v_{11}

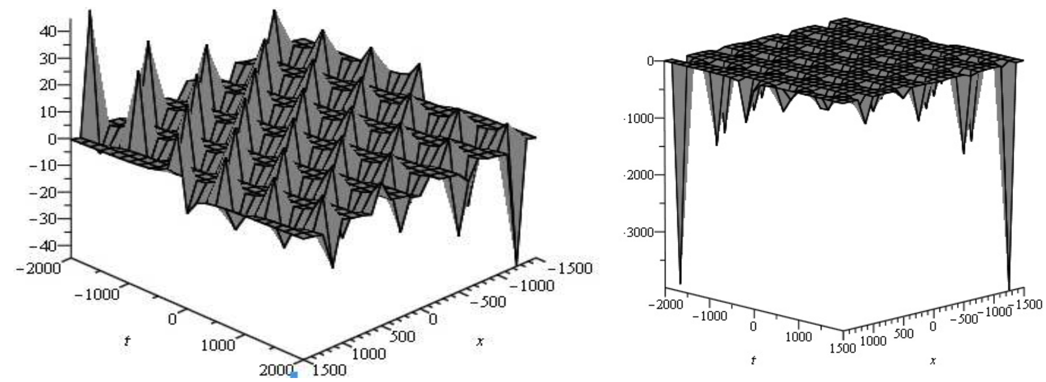

Fig. 4. (a) Graphics of the periodic solution u_{12} **(b) Graphics of the periodic solution** v_{12}

3.3 The Rational Solutions to Maccari's System

(1) When $A=B=0,\ C\neq 0,$ from Table 1, $F(\xi)=-\dfrac{1}{C\xi+\eta}$ (η is an arbitrary constant). By (16), the exact solution to equation (7) is obtained as:

$$u_{21}(x,y,t)=-C\sqrt{2(k\gamma+k^2l)}\frac{e^{i(\alpha x+\beta y+\gamma t)}}{C(k(x+ly-\lambda t))+\eta},$$

$$v_{21}(x,y,t)=-\frac{k}{\gamma+kl}u_{21}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{37}$$

where C is real constants, $\alpha,\beta,\gamma,k,l\ and\ \lambda$ are arbitrary constants.

(2) When $C=B=0$ and A is an arbitrary constant, from Table 1, $F(\xi)=A\xi.$ By (15), the exact solution to equation (7) it can be obtained that:

$$u_{22}(x,y,t)=\frac{\sqrt{\frac{1}{2}(k\gamma+k^2l)}}{k(x+ly-\lambda t)}e^{i(\alpha x+\beta y+\gamma t)},$$

$$v_{22}(x,y,t)=-\frac{k}{\gamma+kl}u_{22}^2(x,y,t)+(\gamma+\alpha^2+\frac{k^2}{2}). \tag{38}$$

where $\alpha,\beta,\gamma,k,l\ and\ \lambda$ are arbitrary constants.

For direct-viewing analysis, we provide the figures of $u_{22}(x,t)$, and $v_{22}(x,t)$, where we choose $l=\beta=0,\ \alpha=-1,\gamma=-2,\ k=-4\ and\ \lambda=1.$

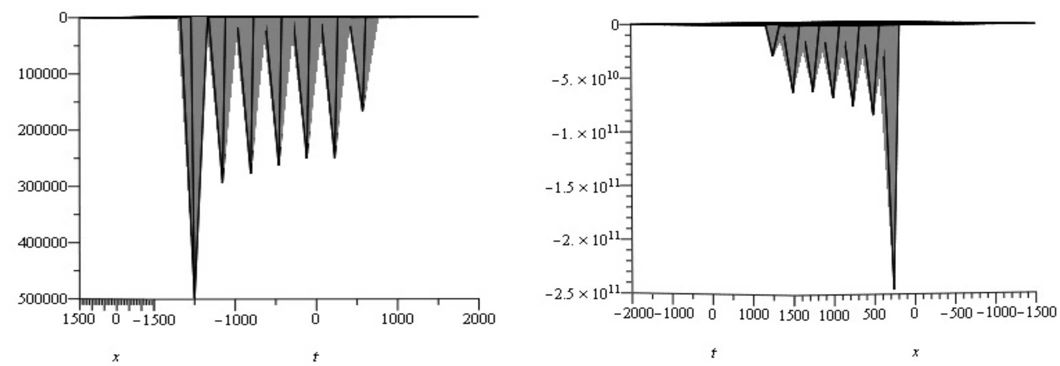

Fig. 5. (a) Graphics of the rational solution u_{22} **(b) Graphics of the rational solution** v_{22}

3.4 The Exponential Solutions to Maccari's System

(1) When $B \neq 0,\ C = 0$ and A is an arbitrary constant, from Table 1, $F(\xi) = \frac{exp(B\xi) - A}{B}$. By (15), the exact solution to equation (7) is found to be:

$$u_{23}(x,y,t) = \frac{AB\sqrt{\frac{1}{2}(k\gamma + k^2 l)}}{exp(Bk(x+ly-\lambda t)) - A} e^{i(\alpha x + \beta y + \gamma t)},$$

$$v_{23}(x,y,t) = -\frac{k}{\gamma + kl} u_{23}^2(x,y,t) + (\gamma + \alpha^2 + \frac{k^2}{2}). \tag{39}$$

where $\alpha, \beta, \gamma, k, l$ and λ are arbitrary constants.

The solutions obtained in this paper are different from solutions in [20], (38) and (39) are almost the same as the some known solutions in [8] except for the coefficients. Other solutions in this investigation of the solutions in [8, 20], will be different.

4 Conclusion

This paper reports on the successful application of the F-expansion method to finding the solutions for Maccari's system. The F-expansion method has been used to get some types of traveling wave solutions including the periodic waves and solitary waves for Maccari's system. It is found that the coupled nonlinear system possesses some other solution structures. we can claim that the F-expansion method can be utilized to solve problems with systems of nonlinear partial differential equations that may arise in the theory of soliton and other related research areas. In the end, it is worthwhile to mention that the proposed method is straightforward and concise and it's applications to other nonlinear physical systems can be investigated in future studies.

Competing Interests

Author has declared that no competing interests exist.

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