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An Interactive Approach for Solving Fuzzy Multi-objective Assignment Problems

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Authors' contributions

This work was carried out in collaboration between both authors. Author HAK formulate the mathematical model, designed the steps of the interactive approach, wrote the first draft of the manuscript and did the analysis of the study. Author MAS wrote the literature review, provided the idea of the problem, checked the numerical results and the whole manuscript. Both authors read and approved the final manuscript.

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Abstract

This paper aims to study fuzzy multi-objective assignment (F-MOAS) problem. The problem is considered by incorporating trapezoidal fuzzy numbers. Through the $\alpha -$ level sets, the problem under consideration is converted into the corresponding ($\alpha -$ MOAS). An interactive approach to improve the weights in the Weighted Tchebysheff program is suggested. Then the stability set of the first kind without differentiability corresponding to the resulted solution is determined. A numerical example is given for illustration.

Keywords: Multi-objective assignment problem; fuzzy numbers; interactive decision making; $\alpha -$ efficient solution; parametric study.

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1 **Introduction**

Assignment problem (AS) is considered a well-studied topic in combinatorial optimization. Also, it is a particular type of the transportation problem (TP), where m tasks (jobs) are to be assigned to an equal number of m machines (workers) in an injective basis so as to minimize (or maximize) the assignment cost (or profit). The assignment problem has a well connection in telecommunication, production planning, economics, etc, Under crisp environment different type of ASPs for a single objective function is discussed in (Swarup et al. [1], and Murthy [2]**)**. A fuzzy assignment model is studied by Chen [3], that's by considering all the individuals involved have the same skills. Mukherjee and Basu [4] solved fuzzy cost assignment problem based on Yager's [5] ranking method that's by transforming the fuzzy assignment into the corresponding deterministic assignment problem. Bao et al. [6] studied multiobjective assignment (MOAS) problem in a fuzzy environment. Kagade and Bajaj [7] applied linear and nonlinear membership functions to solve the MOAS problem. Lin and Wen [8] solved the assignment problem with fuzzy interval cots based on labeling algorithm. Belacela and Boulasselb [9] studied multi- criteria fuzzy assignment problem. Emrouznejad et al. [10] developed an interactive formulation for the fuzzy assignment based on Data Envelopment Analysis (DEA). Kumar and Gupta [11] used different membership functions with Yager's ranking index, [5] through the developed solution method for solving fuzzy assignment problem and fuzzy travelling salesman problems. Pramanik and Biswas [12] studied multi- objective assignment problem with fuzzy trapezoidal fuzzy numbers in the costs, time, and ineffectiveness. Le [13] studied random assignment problem with fractional endowments. Pramanik and Roy [14] applied priority based fuzzy goal programming approach for solving the normal trapezoidal fuzzy multi- objective assignment problem. De and Yadav [15] proposed an algorithm for solving multi- objective assignment problem based on an interactive fuzzy goal programming approach. Abd El-wahed and Lee [16] introduced an interactive fuzzy goal programming approach to determine the best preferred compromise solution for the multi- objective transportation problem with linear membership function. Gao and Lin [17] solved multi- objective transportation with linear and nonlinear parameters based on a two- phase fuzzy goal programming technique. Zimmerman [18] developed suitable membership functions to solve linear programming problem with several objective functions. Verma et al. [19] applied fuzzy programming approach with some nonlinear membership functions to solve a multi- objective transportation problem. Ammar and Khalifa [20] studied multi-objective solid transportation problem with fuzzy parameters both in the objective functions and constraints. Tailor and Dhodiya [21], solved interval- valued multi- objective assignment problem using a genetic algorithm based estimation theory. Tapken et al. [22] proposed a direct solution approach for solving the multi- objective generalized assignment problem with fuzzy numbers both in the objective functions coefficients and the right-hand side of the constraints. Vinoliah and Ganesan [23] proposed a new method for solving fuzzy assignment problem with octagonal fuzzy numbers. The concept of $\alpha-$Pareto optimality of fuzzy parametric programs is introduced by Sakawa and Yano [24]. In his earlier work, Osman [25] introduced the notions of the solvability set, and the stability set of the first kind. Kagade and Bajaj [7] introduced a solution procedure for solving interval- valued multi- objective assignment problem. Lin and Wen [8] studied the assignment problem with imprecise costs.

The rest of the paper is as follows: In section 2, a fuzzy multi- objective assignment (F- MOAS) problem is investigated as specific definition. In section 3, an interactive approach is suggested for solving the $\alpha-$ MOAS problem. The stability set of the first kind corresponding to the obtained solution is determined in section 4. A numerical example is given for illustration in section5. Finally, some concluding remarks are reported in section 6.

2 Problem Formulation and Solution Concepts

Consider the following fuzzy assignment problem:

(F-MOAS) $$\min \tilde{Z}_r(x,\tilde{c}^r) = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}^r x_{ij}, \quad r = 1,2,3,...,l$$

Subject to

$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,...,n$ (Only one person should be assigned the jth job)

$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,...,n$ (Only one job is done by the ith person)

$x_{ij} = 0\, or\, 1.$

Where, $x_{ij}\,(i = j = 1,2,...,n)$ denotes that jth job is to be assigned to the ith person, $\tilde{c}_{ij}^{r0}\,(i = j = 1,2,...,n; r = 1,2,...,l)$ represent fuzzy parameters coefficients. These fuzzy parameters are characterized by fuzzy numbers.

Definition 1. (Fuzzy efficient solution). A point $x^* \in X$ (X is the feasible region) is said to be fuzzy efficient solution of the (F-MOAS) problem if $\tilde{Z}_r\left(x^*,\left(\tilde{c}^r\right)^*\right) \lesseqgtr \tilde{Z}_r\left(x,\tilde{c}^r\right)$ with $\tilde{Z}_r\left(x^*,\tilde{c}^r\right) < \tilde{Z}_r\left(x,\tilde{c}^r\right)$ holds for at least one $r = 1,2,...,l.$

Definition 2. The $\alpha -$level set of the fuzzy numbers $\tilde{c}_{ij}^r$ is defined as the ordinary set $L_\alpha(\tilde{c}_{ij}^r)$ for which the degree of their membership functions exceeds the level α :

$$L_\alpha(\tilde{c}) = \left\{c : \mu_{\tilde{c}_{ij}^r}(c_{ij}^r) \geq \alpha,\ i = j = 1,2,...,n;\ r = 1,2,...,l\right\}$$

For a certain degree of α , the (F-MOAS) problem can be written as in the following non fuzzy form (Rockefeller [26])

(α MOAS) $$\min Z_r(x,c^r) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^r x_{ij}, \quad r = 1,2,3,...,l$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1; \forall j$$

$$\sum_{j=1}^{n} x_{ij} = 1; \forall i$$

$$x_{ij} = 0\, or\, 1,\ c_{ij}^r \in L_\alpha(\tilde{c}_{ij}^r),\quad i = j = 1,2,...,n;\ r = 1,2,...,l.$$

Definition 3. A point x_α^* is called $\alpha -$ Pareto optimal solution to the (α MOAS) problem, if and only if there does not exist another $x,\ c_{ij}^r \in L_\alpha(\tilde{c}_{Iij}^r)$ such that: $Z_r(x,c_{ij}^r) \leq Z_r(x_\alpha^*, c_{ij}^{r^*}), i,j = 1,,...,n;$ $r = 1,,...,l,$ with strict inequality holding for at least one r , where the corresponding values of parameters $c_{ij}^{r^*}$ are called $\alpha -$ level optimal parameters.

The (α MOAS) problem can be resolved by using Weighting Tchebysheff problem

$$\min_{x} \max_{1\le r\le l} \left\{ w_r \left(Z_r(x,c^r) - Z_r^*\right):\ c \in L_\alpha(\widetilde{c})\right\} \tag{1}$$

Or equivalently,

$$\min \left\{\lambda : w_r \left(Z_r(x,c^r) - Z_r^*\right) \le \lambda, r = 1,2,...,l;\ x \in X, c \in L_\alpha(\widetilde{c})\right\} \tag{2}$$

Where, $w_r \ge 0, r = 1, ..., l,\quad 0 \le \lambda_r \in R$, and Z_r^* are the reference point.

3 The Stability Set of the First Kind

In this section, the stability set of the first kind without differentiability is determined. By applying the following conditions:

$$\zeta_{ij}^r (c^{r*} - d_{ij}^r) = 0,$$
$$\xi_{ij}^r (d_{ij}^r - c^{r*}) = 0,$$
$$\zeta_{ij}^r, \xi_{ij}^r \ge 0, i = j = 1,2,...,n;\quad r = 1,2,...,l$$

Where, $\left[\left(d_{ij}^r\right)^1, \left(d_{ij}^r\right)^2\right] = L_\alpha(\widetilde{c}^r), r = 1,2,...,l.$

Consider the following three cases:

Case 1: $\zeta_{ij}^r > 0,\ r \in I_1 \subset \{1,2,...,l\}, \zeta_{ij}^r = 0, r \notin I_1, i = j = 1,2,...,n.$
$\xi_{ij}^r > 0,\ r \in I_2 \subset \{1,2,...,l\}, \xi_{ij}^r = 0, r \notin I_2.$

Let M be the set of all proper subsets of $\{1,2,...,l\}$. Then $S_{I_1,I_2}(x^*,c^*)$ is given by:

$$S_{I_1,I_2}(x^*,c^*) = \left\{ \begin{array}{l} \left(\left(d_{ij}^r\right)^1, \left(d_{ij}^r\right)^2\right) \in R^{2l} : \left(d_{ij}^r\right)^2 = c^{r*},\ r \in I_1,\ \left(d_{ij}^r\right)^2 \ge c^{r*}, r \notin I_1;\ \left(d_{ij}^r\right)^1 = c^{r*}, r \in I_2, \\ \left(d_{ij}^r\right)^1 \le c^{r*}, r \notin I_2 \end{array} \right\}. \tag{3}$$

Hence,

$$S_1(x^*,c^*) = \bigcup_{I_1, I_2 \in M} S_{I_1,I_2}(x^*,c^*). \tag{4}$$

Case 2: $\zeta_{ij}^r, \xi_{ij}^r = 0, r = 1,2,...,l; i = j = 1,2,...,n.$

Then $S_2(x^*,c^*)$ is given by:

$$S_2(x^*,c^*) = \left\{\left(\left(d_{ij}^r\right)^1, \left(d_{ij}^r\right)^2\right) \in R^{2l} : \left(d_{ij}^r\right)^2 \ge c^{r*}, r = 1,2,...,l; \left(d_{ij}^r\right)^1 \le c^{r*}, r = 1,2,...,l\right\} \tag{5}$$

Case 3: $\zeta_{ij}^{1},\ \xi_{ij}^{r} > 0, r = 1,2,...,l; i = j = 1,2,...,n.$

Then $S_3(x^*, c^*)$ is given by:

$$S_3(x^*, c^*) = \left\{\left((d_{ij}^{r})^1, (d_{ij}^{r})^2\right) \in R^{2l} : (d_{ij}^{r})^2 = c^{r*}, r = 1,2,...,l; (d_{ij}^{r})^1 = c^{r*}, r = 1,2,...,l\right\} \quad (6)$$

Thus $S(x^*, c^*) = \bigcup_{q=1}^{3} S_q(x^*, c^*).$ (7)

4 Interactive Approach

In this section an interactive approach to solve the (F-MOAS) problem is introduced as in the following steps:

Step 1: Ask the decision maker (DM) to specify the initial value of $\alpha (0 < \alpha < 1)$ to formulate the problem (α MOAS).

Step 2: Find Z_r^* by solving the following problem

(P$_1$) $$\max_{r=1,2,...,l} \eta_r$$

Subject to

$$Z_r(x, c^r) \geq \eta_r,$$
$$x \in X,\ c \in L_\alpha(\tilde{c}^r), r = 1, ..., l; \eta_r \in R.$$

Step 3: Given an initial reference point. DM provides an initial reference point $\overline{Z}_r^0$ such that

$\overline{Z}_r^0 > Z_r^*$.. Let $J = \{1, 2, ..., l\},\ J^0 = J,\ h = 0.$

Step 4: Search for an $\alpha -$ Pareto optimal solution. Let $\overline{w}_r = \left(\overline{Z}_r^h - Z_r^*\right)^{-1}, r = 1,2,...,l,$ solve

the Weighting Tchebysheff problem (2) at $h -$ iteration .

(P$_2$) $$\min \lambda$$

Subject to

$$w_r\left(Z_r(x, c^r) - Z_r^*\right) \leq \lambda,\ r \in J^h,$$

$$x \in X,\ \mu_{\tilde{c}^r}(c^r) \geq \alpha_p^h, p \in J^h.$$

Where $J^h = J^0 \setminus \{p\}$, and solve to obtain $\alpha -$ Pareto optimal solution $\left(x^{*h},\ c^{*h}\right)$

Step 5: Find the set of parameters $S(x^{*^h}, c^{*^h})$ corresponding to $\left(x^{*^h},\ c^{*^h}\right)$ from the (3)-(7).
Step 6: Determine the termination. Ask the DM to compare

$\left(Z_1(x^{*^h}, c^{*^h}), Z_2(x^{*^h}, c^{*^h}), ..., Z_l(x^{*^h}, c^{*^h})\right)$ with $\left(Z_1^*,\ Z_2^*, ..., Z_l^*\right)$, then there exists two cases:

(a) If the DM is satisfied with the current Pareto optimal solution, go to step 8 and stop- the best compromise solution is found.
(b) If there is no satisfactory objective and level of the Pareto optimal solution, go to step8 and stop- no best compromise solution is found by this approach.

Step 7: Modify the reference point.

(i) The DM chooses g_h in J^h such that Z_{g_h} is an unsatisfactory objective in $\left\{Z_r : r \in J^h\right\}$ at $Z_r(x^h, c^{r^h})$. Let $J^{h+1} = J^h \setminus \{g_h\}$. Separate J^{h+1} into the following two parts:

Part1: $J_1^h = \Big\{ r \in J^{h+1} : Z_r(x^k, c^{r^h}) < \overline{Z}_r^h$, and DM wishes to release the value of Z_r at $Z_r(x^h, c^{r^h}) \Big\}$.

Part2: $J_2^h = J^{h+1} / J$.

(ii) For $r \in J_i^h$, the DM introduces Γ_r^h the amount to be relaxed for Z_r at $Z_r(x^h, c^{r^h})$ such that $\Gamma_r^h \in \left] 0, \left(\overline{Z}_r^h - Z_r x^h, c^{r^h}\right)\right]$ Let $\overline{Z}_r^{h+1} = Z_r(x^h, c^{r^h}) + \Gamma_r^h$. For $r \in J_2^h$, let $i \in J_2^h$, and $\overline{Z}_r^{h+1} = Z_r(x^h, c^{r^h})$. For $r \in J^h / J^{h+1}$, let $\overline{Z}_r^{h+1} = \overline{Z}_r^h$.
(iii) In the case that $\overline{Z}_r^{h+1} = Z_r(x^h, c^{r^h})$ for all $r \in J^h \setminus \{g_h\}$, return to (i) to separate J^{h+1} again or to (ii) to increase the amount to be relaxed for some $Z_r(r \in J_1^h)$ at $Z_r(x^h, c^{r^h})$, if the DM wishes to do so. Otherwise, stop and there is not satisfactory $\alpha -$ Pareto optimal solution. In the case that $\overline{Z}_r^{h+1} \neq Z_r(x^h, c^{r^h})$ for some $r \in J^h \setminus \{g_h\}$, go to (iv).
(iv) Let $g = g_h, Z_r^{'} = \overline{Z}_r^{h+1}, r = 1, ..., l, r \neq g_h$, and solve the following auxiliary problem

(P$_3$)
$$\min Z_g(x, g_h)$$
Subject to
$$Z_r(x, c^r) \le Z_r^{'},\ r \neq g,$$
$$x \in X,\ c \in L_\alpha(\widetilde{c}^{\,r}), r = 1, 2, ..., l.$$

Let $(x^{'^h}, c^{'^h})$ be an optimal solution. When $Z_{g_h}(x^{'^h}, c_{g_h}^{'^h}) = Z_{g_h}(x^h, c_{g_h}^h)$ or $Z_{g_h}(x^{'^h}, c_{g_h}^{'^h})$ for objective Z_{g_h} is not satisfactory to the DM, return to (ii) to increase the amount to be relaxed for some $Z_r(r \in J_1^h)$ at $Z_r(x^h, c^{r^h})$ if the DM wishes to do so. Otherwise, go to step 8, there is no satisfactory

$\alpha-$ Pareto optimal solution. When $Z_{g_h}(x^{*h}, c^{*h}_{g_h}) \neq Z_{g_h}(x^h, c^{r^h}_{g_h})$ and $Z_{g_h}(x^{*h}, c^{*h}_{g_h})$ for objective Z_{g_h} is satisfactory to the DM (he/ she) provides $\Gamma^h_{g_h}$, the largest amount to be improved for Z_{g_h} at $Z_r(x^h, c^{r^h})$, such that $\Gamma^h_{g_h} \in \left]0, \left(Z_{g_h}(x^h, c^{r^h}_{g_h}) - Z_{g_h}(x^{*h}, c^{*h}_{g_h})\right)\right]$. Let $\overline{Z}^{h+1}_{g_h} = Z_{g_h}(x^h, c^h_{g_h}) \setminus \Gamma^h_{g_h}$.

(v) If $\overline{Z}^{h+1}_{g_h} < Z_{g_h}(x^{*h}, c^{*h}_{g_h})$, let $h = h+1$ and return to step (iii). Otherwise, let $(x^{h+1}, c^{h+1}) = (x^{*h}, c^{*h})$, let $h = h+1$, and return to step (iv) when (x^{*h}, c^{*h}) is an unique optimal solution of (P_3) or let (x^{*h}, c^{*h}) be an optimal solution of the problem(2). Let $h = h+1$, and return to step (iii). If $\overline{Z}^{h+1}_{g_h} \geq Z_{g_h}(x^{*h}, c^{*h}_{g_h})$, let $h = h+1$ and return to step 4.

Step 8: Stop.

5 Numerical Example

Consider the following problem

$$\min Z_r(x, c^r) = \sum_{i=1}^{3}\sum_{j=1}^{3} c^r_{ij}\, x_{ij}, \quad r = 1,2$$

Subject to

$$\sum_{i=1}^{3} x_{ij} = 1; \forall j$$

$$\sum_{j=1}^{3} x_{ij} = 1; \forall i$$

$$x_{ij} = 0 \, or 1.$$

With membership functions

$$\mu_{\tilde{c}^r_{ij}}(c^r_{ij}) = \begin{cases} 0, & -\infty < c^r_{ij} \leq \left(c^r_{ij}\right)^1, \\ 1 - \left(\dfrac{c^r_{ij} - \left(c^r_{ij}\right)^2}{\left(c^r_{ij}\right)^1 - \left(c^2_{ij}\right)^2}\right), & \left(c^r_{ij}\right)^1 \leq c^r_{ij} \leq \left(c^r_{ij}\right)^2, \\ 1, & \left(c^r_{ij}\right)^2 \leq c^r_{ij} \leq \left(c^r_{ij}\right)^3,\ i = j = 1, \\ 1 - \left(\dfrac{c^r_{ij} - \left(c^r_{ij}\right)^3}{\left(c^r_{ij}\right)^4 - \left(c^r_{ij}\right)^3}\right), & \left(c^r_{ij}\right)^3 \leq c^r_{ij} \leq \left(c^r_{ij}\right)^4, \\ 0, & \left(c^r_{ij}\right)^4 \leq c^r_{ij} < \infty \end{cases}$$

Where,

$c^1_{11} = (7,9,10,11)$, $c^1_{12} = (5,7,8,10)$, $c^1_{13} = (12,14,15,17)$, $c^1_{21} = (10,12,13,15)$,$c^1_{22} = (9,11,12,14)$, $c^1_{23} = (10,12,13,15)$,$c^1_{31} = (5,7,8,10)$, $c^1_{32} = (7,9,10,11)$, $c^1_{33} = (6,8,9,11)$, $c^2_{11} = (10,12,13,15)$, $c^2_{12} = (12,14,15,17)$,$c^2_{13} = (5,7,8,10)$, $c^2_{21} = (7,9,10,11)$, $c^2_{22} = (16,19,20,22)$, $c^2_{23} = (9,11,12,14)$, $c^2_{31} = (12,14,15,17)$, $c^2_{32} = (7,9,10,11)$, $c^2_{33} = (9,11,12,14)$

Step 1: Take $\alpha = 0.5$, then

$$8 \le c_{11}^1 \le 10.5,\ 6 \le c_{12}^1 \le 9,\ 13 \le c_{13}^1 \le 16,\ 11 \le c_{21}^1 \le 14,\ 10 \le c_{22}^1 \le 13,\ 11 \le c_{23}^1 \le 14,$$
$$6 \le c_{31}^1 \le 9,\ 8 \le c_{32}^1 \le 10.5,\ 7 \le c_{33}^1 \le 10,\ 11 \le c_{11}^2 \le 14, 13 \le c_{12}^2 \le 16, 6 \le c_{13}^2 \le 9,$$
$$8 \le c_{21}^2 \le 10.5,\ 17.5 \le c_{22}^2 \le 21,\ 10 \le c_{23}^2 \le 13,\ 13 \le c_{31}^2 \le 16,\ 8 \le c_{32}^2 \le 10.5, 10 \le c_{33}^2 \le 13.$$

Then the non-fuzzy (α MOAS) problem is

$$\begin{aligned}\min Z_1(x,\widetilde{c}) = {} & [8,10.5]x_{11} + [6,9]x_{12} + [13,16]x_{13} + [11, 14]x_{21} + [10, 13]x_{22} + [11,14]x_{23} \\ & + [6, 9]x_{31} + [8,10.5]x_{32} + [7,10]x_{33} \\ \min Z_2(x,\widetilde{c}) = {} & [11, 14]x_{11} + [13, 16]x_{12} + [6, 9]x_{13} + [8, 10.5]x_{21} + [17.5, 21]x_{22} + [10, 13]x_{23} \\ & + [13,16]x_{31} + [8,10.5]x_{32} + [10,13]x_{33}\end{aligned}$$

Subject to

$$\begin{aligned}&\sum_{i=1}^{3} x_{ij} = 1, \\ &\sum_{j=1}^{3} x_{ij} = 1, \\ &x_{ij} = 0\, or\, 1 (i = j = 1,2,3).\end{aligned}$$

Step 2: Solve the problem (P_1) to get Z_r^* as

$$\max_{r=1,2} \ \eta_r$$

Subject to

$$\begin{aligned}&10 x_{11} + 8 x_{12} + 15 x_{13} + 13 x_{21} + 12 x_{22} + 13 x_{23} + 8 x_{31} + 10 x_{32} + 9 x_{33} \ge \eta_1 , \\ &13 x_{11} + 15 x_{12} + 8 x_{13} + 10 x_{21} + 20 x_{22} + 12 x_{23} + 15 x_{31} + 10 x_{32} + 1\widetilde{2} x_{33} \ge \eta_2 , \\ &\sum_{i=1}^{3} x_{ij} = 1, \\ &\sum_{j=1}^{3} x_{ij} = 1, \\ &x_{ij} = 0\ or\ 1 (i = j = 1, 2, 3). \\ &\eta_r \in R, r = 1, 2.\end{aligned}$$

The solution is

$$x^* = (0,1,1,1,0,1,1,1,0), c^{1^*} = (10,8,15,13,12,13,8,10,9), c^{*^2} = (13,14,8,10,20,12,15,10,12),$$
$$Z_1^* = Z_2(x^*, c^*) = 67 \qquad , \quad Z_2^* = Z_2(x^*, c^*) = 70.$$

Step 3: Ask the DM to provide an initial reference point $\overline{Z}_r^0$ such that $\overline{Z}_r^0 > Z_r^*$.

$$\overline{Z}_1^0 \in (55, 72.5]? \quad 69 \qquad \overline{Z}_2^0 \in (58, 75]? \quad 72. \qquad \overline{Z}^0(69,\ 72)^T.$$

Step 4: Find $\overline{w}_1 = \dfrac{1}{2}$, $\qquad \overline{w}_2 = \dfrac{1}{2}$.

Solve the following problem

$$\min \lambda$$

Subject to

$$\begin{pmatrix} 23x_{11} + 23x_{12} + 23x_{13} + 23x_{21} + 32x_{22} + 25x_{23} \\ + 23x_{31} + 20x_{32} + 21x_{33} - 2\lambda \end{pmatrix} \le 137 \ ,$$

$$\sum_{i=1}^{3} x_{ij} = 1,\ j = 1,2,3,$$

$$\sum_{j=1}^{3} x_{ij} = 1,\ i = 1,2,3,$$

$$x_{ij} = 0 \ or\ 1\,(i = j = 1,2,3),$$

$$0 \le \lambda \in R,$$

The solution is

$$x_{11}^{*0} = x_{12}^{*0} = x_{13}^{*0} = x_{23}^{*0} = x_{32}^{*0} = x_{33}^{*0} = 0,\ x_{13}^{*0} = x_{22}^{*0} = x_{31}^{*0} = 1,$$
$$\lambda^* = 0.8, Z_1^* \subset [29,38],\ Z_2^* \subset [36.5,46].$$

The corresponding fuzzy objectives value are: $Z_1 = (26,32,35,41),\ Z_2 = (34,40,43,49).$

Is the solution satisfactory to the DM ?Yes. The stability set of the first kind corresponding to $\left(x^{*0};\ c^{*0}\right)$ is:

$$\zeta_{13}^1(c^{*1} - d_{13}^1) = 0,$$
$$\zeta_{22}^1(c^{*1} - d_{22}^1) = 0,$$
$$\zeta_{31}^1(c^{*1} - d_{31}^1) = 0,$$
$$\xi_{13}^2(c^{*2} - d_{13}^2) = 0,$$
$$\xi_{22}^2(c^{*2} - d_{22}^2) = 0,$$
$$\xi_{31}^2(c^{*2} - d_{31}^2) = 0 \ ;\zeta_{ij}^r, \xi_{ij}^r \ge 0, r = 1,2; i = j = 1,2,3.$$

We have $I_1 \subseteq \{1,2\}$. For $I_1 = \phi,\ \zeta_{13}^1 = \zeta_{22}^1 = \zeta_{31}^1 = 0, \xi_{13}^2 = \zeta_{22}^2 = \zeta_{31}^2 = 0$. Then

$$S_{I_1}(x^{*0};\ c^{*0}) = \begin{Bmatrix} d_2 \in R^6 : 8 \le d_{13}^1 \le 16, 10 \le d_{22}^1 \le 13, 6 \le d_{31}^1 \le 9,\ 6 \le d_{13}^2 \le 9 \\ ,17.5 \le d_{22}^2 \le 21, 13 \le d_{31}^2 \le 16 \end{Bmatrix}.$$

For $I_2 = \{1\}$, $\zeta_{ij}^r > 0$, $\xi_{ij}^r = 0$. Then

$$S_{I_2}(x^{*0}; c^{*0}) = \left\{ d_2 \in R^6 : \; d_{13}^1 = 13, d_{22}^1 = 10, d_{31}^1 = 9, 6 \le d_{13}^2, 17.5 \le d_{22}^2, 13 \le d_{31}^2 \right\}.$$

For $I_3 = \{2\}$, $\zeta_1 = 0$, $\xi_2 > 0$. Then

$$S_{I_3}(x^{*0}; c^{*0}) = \left\{ d_2 \in R^6 : 13 \le d_{13}^1, 10 \le d_{22}^1, 6 \le d_{31}^1, d_{13}^2 = 6, d_{22}^2 = 17.5, d_{31}^2 = 13 \right\}.$$

For $I_4 = \{1, 2\}$, $\zeta_1 > 0$, $\xi_2 > 0$. Then

$$S_{I_4}(x^{*0}; c^{*0}) = \left\{ d_2 \in R^6 : 13 = d_{13}^1, 10 = d_{22}^1, 6 = d_{31}^1, d_{13}^2 = 6, d_{22}^2 = 17.5, d_{31}^2 = 13 \right\}.$$

Thus, $S(x^{*0}, c^{*0}) = \bigcup_{q=1}^{4} S_{I_q}(x^{*0}, c^{*0}).$

6 Conclusions

In this paper, a fuzzy multi- objective assignment (F-MOAS) problem has been investigated. The advantages of the fuzzy is that the problem with fuzzy allows the DM to deal with a situation realistically. An interactive approach to improve the weights in the Weighted Tchebysheff program has been suggested. The stability set of the first kind without differentiability corresponding to the obtained solution has been determined.

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Competing Interests

Authors have declared that no competing interests exist.

References

[1] Swarup K, Gupta PK, Mohan M. Operations research, 11th Ed., Sultan Chand & Sons, New Delhi; 2003.

[2] Murthy PR. Operations research, 2nd Ed. New Age International Limited, New York; 2007.

[3] Chen MS. On a fuzzy assignment problem solid transportation problem. Tamkang Journal. 1985;22: 407-411.

[4] Mukherjee S, Basu K. Application of fuzzy ranking method for solving assignment problem with fuzzy costs. International Journal of Computational and Applied Mathematics. 2010;5(3):359-368.

[5] Yager RR. A procedure for ordering fuzzy subsets of the unit interval. Information Sciences. 1981;24: 143-161.

[6] Bao CP, Tsai CM, Tsai M. A new approach to study the multi- objective assignment problem. WHAMPOA- An Inter Disciplinary Journal. 2007;53:123-132.

[7] Kagade KL, Bajaj VH. Fuzzy approach with linear and non- linear membership functions for solving multi- objective assignment problems. Advances in Computational Research. 2009;1(2):14-17.

[8] Lin CJ, Wen UP. A labeling algorithm for the fuzzy assignment problem. Fuzzy Sets and Systems. 2004;142:373-391.

[9] Belacela N., and Boulasselb, M.R. (2001). Multi criteria fuzzy assignment problem: A useful tool to assist medical diagnosis. Artificial Intelligence in Medicine, (21): 201- 207.

[10] Emrouznejad A, Angiz AMZ, Ho WL. An alternative formulation for the fuzzy assignment problem. European Journal of Scientific Research Society. 2012;63:59-63.

[11] Kumar A, Gupta A. Methods for solving fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions. Fuzzy Information and Engineering. 2011;3(1):3-21.

[12] Pramanik S, Biswas P. Multi- objective assignment problem with generalized trapezoidal fuzzy numbers. International Journal of Applied Information Systems. 2012;2(6):13-20.

[13] Le P. Competitive equilibrium in the random assignment problem. International Journal of Economic Theory. 2017;13(4):369-385.

[14] Pramanik S, Roy TK. A fuzzy goal programming technique for solving multi- objective transportation problem. Tamsui Oxford Journal of Management Sciences. 2008;22:67-89.

[15] De KP, Yadav B. An algorithm to solve multi- objective assignment problem using interactive fuzzy goal programming approach. Int. J. Contemp. Math. Sciences. 2011;6(34):1651-1662.

[16] AbdEl-wahed WF, Lee MS. Interactive fuzzy goal programming for multi- objective transportation problems. Omega. 2006;34:158-166.

[17] Gao PS, Liu AS. Two phase fuzzy algorithms for multiobjective transportation problem. Journal of Fuzzy Mathematics 2004;12:147-155.

[18] Zimmermann JH. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems. 1978;1:45-55.

[19] Verma R, Biswal PW, Biswas A. Fuzzy programming technique to solve multi-objective transportation problem with some nonlinear membership functions. Fuzzy Sets and Systems. 1997;91: 37-43.

[20] Ammar EE, Khalifa AH. Study on multiobjective solid transportation problem with fuzzy numbers. European Journal of Scientific Research. 2015;125(1):7-19.

[21] Tailor AR, Dhodiya JM. A genetic algorithm based hybrid approach to solve multi-objective interval assignment problem by estimation theory. Indian Journal of Science& Technology. 2016;9(35):1-13.

[22] Tapkan P, Ozbakir L, Baykasoglk A. Solving fuzzy multiple objective generalized assignment problems directly via bees algorithm and ranking. Expert Systems with Applications. 2013;40(3): 892-898.

[23] Vinoliah EM, Ganesan K. Fuzzy optimal solution for a fuzzy assignment problem with octagonal fuzzy numbers. National Conference on Mathematical Techniques and its Applications (NCMTA18), IOP Publishing; 2018.

[24] Sakawa M, Yano H. Interactive decision making for multi objective nonlinear with fuzzy parameters. Fuzzy Sets and Systems. 1989;29:315-326.

[25] Osman M. Qualitative analysis of basic notions in parametric convex programming. II (Parameters in the objective function). Appl. Mat. 1977;22:333-348.

[26] Rockafellar R. Duality and stability in external problems involving convex functions. Pacific Journal of Mathematics. 1967;21:167-181.

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