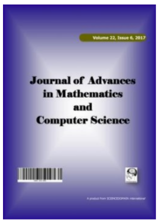

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An Optimal Media Selection Model for a Company in Ghana

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Authors' contributions

This work was carried out in collaboration between all authors. Author JAP designed the study and fine tune the resulting model. Author KH carried out the model and the numerical simulation of the study. Author RO assisted to fine tune the model, managed the literature searches and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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Abstract

Selection of media and budget allocation is a major concern in advertising. It involves choosing the appropriate media that is effective in reaching the target audience of the population in consideration. Generally, predicting an optimal target audience exposure in selection of media is a complex problem. In this study, a Linear Programming technique is used to investigate the budgetary allocation of a Company for effective media selection planning. The problem is formulated using empirical data from the company and other media sources. The resulting linear programming model is analyzed using Quantitative Manager for Business version 3.2 and Linear Programming Solver version 5.2.2 which embodies the simplex algorithm method. The results show that the optimal target audience is 635,048,700. The optimum media mix which attracts

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a significant audience exposure and generates the desired objective value for the advertising campaign include; three (3) Television media outlet, two (2) print media outlet, and ten (10) radio media outlet.

Keywords: Media Selection; linear programing; simplex algorithm; basic solution; Feasible set; Convex/Concave set.

1 Introduction

Currently, the media selection planning has become increasingly complex which is evident in the development of a variety of methods of solution [1], [2], [3], which according to Broadbent et al. [4], depends on the assumptions made in each case. Thus, media selection planning has attracted a wide variety of model building than any other problem in the marketing domain [5]. For example advocates of the linear programming approach to the media selection and planning problem would favour the following problem statement offered by Day [6]: "Media selection problem is to allocate a scarce resource among a large number of alternatives so that the best possible contribution is made to a central objective". However the majority of approaches to the media problem suffer in that they are static models apportioning the media budget to alternative media in order to maximize 'effectiveness' at some point in time.

Media selection using a conceptual model is considered a serious challenge in Ghana. The fact is that most companies are more inclined to very little or no conceptual approach in dealing with most media campaigns. In some instances, it is considered as an insurmountable challenge in the country [7]. It is in the light of this that the linear programming conceptual model is adopted to expose the clear cut benefits of the conceptual approach [8]. The reason being that, conceptual models allow for optimization of the media selection planning in such a way that, the effect of each variable present in relation to its cost and effect which is completely absent in heuristic approach adopted over the years in Ghana is addressed.

Manufacturers and other marketing companies carry out advertisement to inform prospective customers about a product or service or to remind them of their existence. It is the hope of the advertiser that his message will make customers buy a product or service who may not otherwise have done so. The advertiser believes that his investment will bring profit or some kind of performance measure to the company. In any of these scenarios, the advertiser buys a space or time in the media such as TV, Billboards, Radio, Newspaper as a channel to transmit the information to the prospective target audiences. They make choices with regards to which category or a combination of media categories to choose from in order to accomplish their desired objective. Besides the decision on the media selection, financial resources available to be spent on advertising is also of a great concern.

Media selection and planning is part of the integrated advertising planning process which began from 1900 - 1960. In online display advertisement, a large-scale Internet Media Selection Optimization problem has been proposed which considers including consumer targeting and target frequency of ad exposure [2]. Stefanos et al. [9] explored the media selection allocation problem to determine the optimum media for a commercial enterprise business entity. Bass and Lonsdale [10] in formulating their linear programming approach stated the problem as one of selecting the best set from among various media alternatives. Chang et al. [11] made their contribution in the field of media scheduling. The aim of the investigation was on the advertising wear out phenomenon. He finally proposed an optimal media mix plan on the advertising pulsation strategy to resolve the problem whilst maximizing the target exposure. Hooshang et al. [3] also developed a hybrid advertising media selection model using AHP and Fuzzy-based GA decision making which integrates qualitative and quantitative models and human-based information.

In this paper, we devote attention to linear programming to examine the media selection planning problem in Ghana.

2 Theoretical Concept of the Simplex Algorithm

Linear Programming (LP) is the problem of maximizing or minimizing a linear function subject to linear constraints [12]. A typical optimization problem is to find the best element from a given set (*feasible set*) [13]. In Linear Programming (LP) problems, we need a criterion called an *objective function* $Z = f(\mathbf{x})$, and the *feasible set*, $\mathbf{x}$ is usually defined by

$$\{\mathbf{x} \in \mathbf{R}^n | g_i(\mathbf{x}) \leq 0, \ \ i = 1, \ldots, m\}.$$

where $g_i(\mathbf{x}) \leq 0$ are *constraint functions*.

2.1 The Standard and Canonical form

In general, the canonical form of an LP problem is expressed as:

$$\begin{aligned} \textit{Maximize} \qquad Z &= \sum_{j=1}^{n} c_j x_j && (2.1) \\ \textit{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j &\leq b_i \qquad i = 1, 2, \ldots, m && (2.2) \\ \textit{and} \qquad\qquad x_j &\geq 0 \qquad j = 1, 2, \ldots, n && (2.3) \end{aligned}$$

The standard form of an LP problem is expressed as:

$$\begin{aligned} \textit{Maximize} \qquad Z &= \sum_{j=1}^{n} c_j x_j && (2.4) \\ \textit{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j &= b_i \qquad i = 1, 2, \ldots, m && (2.5) \\ \textit{and} \qquad\qquad x_j &\geq 0 \qquad j = 1, 2, \ldots, n && (2.6) \end{aligned}$$

In many situations, the variables, x_j may be unrestricted in sign. As a result LP problems with unrestricted variables is converted to an equivalent problem having non-negative variables. This is achieved by expressing each unrestricted variable as the difference of two non-negative variables as:

$$x_j = x_j' - x_j'' \qquad\qquad x_j', x_j'' \geq 0$$

where x_j' and x_j'' determines the value of x_j and must both appear in the objective or constraint function [14], [15].

2.2 Matrix Representation of Standard LP Problem

In the matrix notation, such an optimization problem can be formulated in the standard form as:

$$\begin{aligned} \textit{Maximize} \quad Z &= \mathbf{c}^T \mathbf{x} && (2.7) \\ \textit{Subject to} \quad \mathbf{Ax} &\leq \mathbf{b} && (2.8) \\ \mathbf{x} &\geq 0 && (2.9) \end{aligned}$$

or in the canonical form as

$$\begin{aligned}
Maximize \quad Z &= \mathbf{c}^T\mathbf{x} && (2.10)\\
Subject\ to \quad \mathbf{Ax} &= \mathbf{b} && (2.11)\\
\mathbf{x} &\geq 0 && (2.12)
\end{aligned}$$

where $\mathbf{c} = (c_1, c_2, \dots, c_n)^T$, $\mathbf{b} = (b_1, b_2, \dots, b_m)^T$, and $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ and $\mathbf{A}$ is $m \times n$ coefficients matrix of rank m. The constraints of an LP problem may come with $\leq$, $=$, or $\geq$ which are converted into equalities by adding *slack* and/or *surplus* variables.

The set of constraints, $\mathbf{Ax} = \mathbf{b}$ of a linear programing problem, where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

is an $m \times n$ matrix of rank m and $n = r + m$ and

$$\mathbf{x} = [x_1, x_2, x_3, \dots, x_n]^T$$

is a linear system of m simultaneous equation in n $(n > m)$ unknowns. We can rewrite $\mathbf{A} = [\mathbf{B}, \mathbf{N}]$ such that $\mathbf{B}$ is an $m \times m$ non-singular matrix and $\mathbf{N}$ an $m \times (n - m)$ matrix and $\mathbf{x} = [\mathbf{x_B}, \mathbf{x_N}]^T$ obtained by partitioning $\mathbf{x}$. Hence the constraints $\mathbf{Ax} = \mathbf{b}$ can be rewritten as:

$$[\mathbf{B}, \mathbf{N}][\mathbf{x_B}, \mathbf{x_N}]^T = \mathbf{b} \tag{2.13}$$

which implies

$$\mathbf{Bx_B} + \mathbf{Nx_N} = \mathbf{b} \quad \text{or} \quad \mathbf{x_B} = \mathbf{B}^{-1}\mathbf{b} - \mathbf{B}^{-1}\mathbf{Nx_N} \tag{2.14}$$

If all the $n - m$ variables not associated with the columns of matrix $\mathbf{B}$ is zero ie., $\mathbf{x_N} = 0$, the resulting system of equations yields the basic solution

$$[\mathbf{x_B}, \mathbf{x_N}]^T = \begin{bmatrix} \mathbf{B}^{-1} \\ \mathbf{0} \end{bmatrix} \tag{2.15}$$

The remaining m variables are called *basic variables* and the associated basic solution referred to as *basic feasible solution* if the variables also satisfy the non-negativity conditions, $\mathbf{x} \geq 0$. Again if these variables satisfy all the constraints $\mathbf{Ax} = \mathbf{b}$, then the solution is known as a *feasible solution* [15].

Theorem 2.1. *A collection of all feasible solutions (if they exist) of an LP problem constitute a convex set.*

Proof. Consider the standard form of an LP problem

$$\begin{aligned}
Maximize \quad Z &= \mathbf{c}^T\mathbf{x}\\
Subject\ to \quad \mathbf{Ax} &= \mathbf{b}\\
\mathbf{x} &\geq 0
\end{aligned}$$

We let S be the collection of all feasible solutions of $\mathbf{Ax} = \mathbf{b}$, $\mathbf{x} \geq 0$. The set S is a convex set if it contains only one element, hence the theorem is true. Now consider two points $\mathbf{x}', \mathbf{x}'' \in S$ such that $\mathbf{x}' \neq \mathbf{x}''$ then

$$\mathbf{Ax}' = \mathbf{b},\ \mathbf{x}' \geq 0 \qquad \text{and} \qquad \mathbf{Ax}'' = \mathbf{b},\ \mathbf{x}'' < 0.$$

Consider again another point $\mathbf{x}'''$ such that

$$\mathbf{x}''' = \lambda \mathbf{x}' + (1-\lambda)\mathbf{x}'', \qquad 0 \leq \lambda \leq .$$

If S is convex then $\mathbf{x}''' \in S$. To show that, we prove that $\mathbf{x}'''$ satisfy $\mathbf{Ax} = \mathbf{b}$. Thus

$$\begin{aligned} \mathbf{Ax}''' &= \mathbf{A}\{\lambda \mathbf{x}' + (1-\lambda)\mathbf{x}''\} \\ &= \lambda \mathbf{Ax}' + (1-\lambda)\mathbf{Ax}'' \\ &= \lambda \mathbf{b} + (1-\lambda)\mathbf{b} = \mathbf{b}, \end{aligned}$$

hence $\mathbf{x}''' \in S$ and the set S is convex, since $\mathbf{x}''' \geq 0$ and satisfy the system $\mathbf{Ax} = \mathbf{b}$. □

Theorem 2.2. *(a) If the convex set of the feasible solutions of the system of equations $\mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq 0$, is a convex polyhedron, then at least one of the extreme points gives an optimal solution.*
(b) If the optimal solution occurs at more than one extreme point, then the value of the objective function will be the same for all convex combinations of these extreme points.

Proof. (a) For the feasible region of the convex polyhedron of the LP problem:

$$\begin{aligned} &Maximize & Z &= \mathbf{c}^T\mathbf{x} \\ &Subject\ to & \mathbf{Ax} &= \mathbf{b} \\ & & \mathbf{x} &\geq 0 \end{aligned}$$

we let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_p$ be the extreme points. Suppose the maximum value of Z occurs at the extreme point $\mathbf{x}'$ among $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_p$. We let $Z^* = \mathbf{c}^T\mathbf{x}'$. Again let Z' be the value of Z at any point $\mathbf{x}'$ of the feasible region. Then $Z' = \mathbf{c}^T\mathbf{x}'$. Since $\mathbf{x}'$ is not an extreme point, there exist scalars $\lambda_1, \lambda_2, \ldots, \lambda_p$ such that:

$$\mathbf{x}' = \lambda_1 \mathbf{x}_1 + \lambda_2 \mathbf{x}_2 + \ldots + \lambda_p \mathbf{x}_p$$

where

$$\sum_{j=1}^{p} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, 2, \ldots, p$$

Substituting $\mathbf{x}'$ in Z', we get:

$$Z' = \mathbf{c}^T\{\lambda_1 \mathbf{x}_1 + \lambda_2 \mathbf{x}_2 + \ldots + \lambda_p \mathbf{x}_p\} \leq \mathbf{c}^T\mathbf{x}^i, \ i.e.\ Z' \leq Z^*$$

This shows that the optimum solution at the extreme point is better than the solution at any other point of the feasible region.

(b) In the feasible region where the objective function has equal and optimum value, we let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ $(k \leq p)$ be the extreme points. Hence we write,

$$Z^* = \mathbf{c}^T\mathbf{x}_1 = \mathbf{c}^T\mathbf{x}_2 = \ldots = \mathbf{c}^T\mathbf{x}_k.$$

We again let

$$\mathbf{x} = \lambda_1 \mathbf{x}_1 + \lambda_2 \mathbf{x}_2 + \ldots + \lambda_k \mathbf{x}_k$$

where

$$\sum_{j=1}^{p} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, 2, \ldots, k$$

Then

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &= \mathbf{c}\{\lambda_1 \mathbf{x}_1 + \lambda_2 \mathbf{x}_2 + \ldots + \lambda_k \mathbf{x}_k\} \\ &= \lambda_1 \mathbf{cx}_1 + \lambda_2 \mathbf{cx}_2 + \ldots + \lambda_k \mathbf{cx}_k \\ &= (\lambda_1 + \lambda_2 + \ldots + \lambda_k)Z^* = Z^* \end{aligned}$$

□

We state the following theorems without proof.

Theorem 2.3. *If a standard LP problem with constraints* $\mathbf{Ax} = \mathbf{b}$, $\mathbf{x} > 0$ *where* $\mathbf{A}$ *is an* $m \times n$ *matrix of rank* m $(\leq n)$ *has a feasible solution, then it also has a basic feasible solution. (See [15])*

Theorem 2.4. *A necessary and sufficient condition for a vector* $\mathbf{x}$ *in a convex set* S *to be an extreme point is that* $\mathbf{x}$ *is a basic feasible solution satisfying the system* $\mathbf{Ax} = \mathbf{b}$, $\mathbf{x} \geq 0$ *(See [15])*

Now, consider a basic feasible solution $\mathbf{x_B} = \mathbf{B}^{-1}\mathbf{b}$, if there exist at least one component of $\mathbf{x_B}$ that is zero, we call the solution degenerate. The basic feasible solution is called *non-degenerate* if all the components of $\mathbf{x_B}$ are non-zero ($\mathbf{x_B} > 0$).

3 Data for the Problem

3.1 Introduction

The selected Company is launching an advertisement promotional campaign called; "The Hit-maker" which is intended to last for three months. The program is aimed at identifying young artists who wanted to become the best musician in the country. Also, the aspirants had the opportunity to take part in a nationwide selection process thus; the "want-to-be musicians" are required to go to some selected radio stations in all the Regional Capitals to sing their songs upon which listeners and viewers will call or text to vote for their favorite. The best in all the regions together with other contestants in the Capital City of Ghana will be invited to the company's headquarters to compete for the final and runner-up positions respectively.

Since this program is targeting all who want to be the best musician in the country and to inform the general public as well, the advertiser needs, to advertise the program to the general public and the prospective target audiences to inform them of the existence of the program. This can only be accomplished by buying space or time on the various media to achieve the advertiser's desired objective. Currently, there are a lot of mass media in the country and the advertiser may not utilize all to maximize the target exposure due to financial constraint. This necessitates that they must opt for some media alternatives to bring about exposure. The advertiser has decided to use some selected state owned and private media to achieve the desired objective. The media are categorized into three major media types viz; TV, Radio, and Newspapers with specific selected media vehicles within each category since prospective target audiences have choices to which media they used to obtain information.

3.2 Data

Data was collected on the media categories which the company intended to use for the advertisements. These are as follows:

1. **Television Media:** GTV, TV3, Metro TV, Viasat 1, and Crystal TV
2. **Print Media:** Daily Graphic, Graphic Showbiz, Ghanaian Business, Junior Graphic, Ghanaian Sports, The Mirror, Daily Guide, The Chronicle, Ghanaian Times, Ninety Minutes
3. **Radio Media:** Adom FM, Hitz FM, Joy FM, Happy FM, Peace FM, Asempa FM, Radio Gold, Citi FM, Unique FM, BBC Radio, Choice FM, Hot FM, Obonu FM, Angel FM, Hello FM, Love FM, Kapital Radio, Nhyira FM, Otec FM, Kessben FM, Radio Capital, Sunrise FM, Kyzz FM, Aseda FM, Melody FM, Volta Star, Savana Radio, Uria Radio, Radio Upper West (UW), Radio Bar. Thus five (5) TV media, ten (10) Print media, and thirty (30) Radio media were used for the adverts.

Budget allocation and the constraints of the budget are outlined as follows;

- The total advertising budget fixed for the promotional campaign was GH¢600,000.00
- The budget allocation to TV media was GH¢200, 000.00
- The maximum budget allocation to the Print media was 25% of the total budget (i.e 150, 000).
- The budget allocation to Radio media was GH¢250,000.00
- The advertiser's policy was that advert should be cast on all the seven days of the week with priority on working days and Saturdays.
- The maximum advert available in any of the broadcast media was 270 throughout the length of the program.
- The advertiser's policy was that the Advertising units on Radio media should be more than that of TV.
- The advertiser's policy was that the Advertising units on Radio media should be more than that of Print.
- The advertiser's policy was that the Advertising units on TV media should be more than that of Print.
- The advertiser's policy was that the maximum budget allocation to Radio media should be more than that of TV.
- The advertiser's policy was that the maximum budget allocation to Radio media should be more than that of Print.
- The budget allocation to TV media should be more than that of print.
- The advertiser expects that the promotional campaign reaches a greater number of audiences, and yet work within its budgetary limit. Therefore, it has estimated that a total of 500 adverts may be cast across all media. The policies and restrictions given above would constitute the major constraints of the model.

Table 1. Cost of advert and estimated audience exposure for graphic media

Print media	Cost (Gh¢) per advert	Estimated Audience Exposure
Daily Graphic	3,928	2,000,000
Graphic Showbiz	970	500, 000
Ghanaian Business	1, 614	900,000
Junior Graphic	812	750,000
Ghanaian Sports	970	700,000
The Mirror	2,213	1,600,000
Daily Guide	2,909	30,000
The Chronicle	1,648	550,000
Ghanaian Times	1, 203	600,000
Ninety Minutes	682	650,000

4 Model Formulation

4.1 The Decision Variables

The decision variables consist of the number of times to use each medium in order to maximize the desired target exposure whilst operating under the restrictions and policies of the company. Since all the media outlets considered in this study fall into three major media categories, the decision

variables are made up of five (5) from TV, ten (10) from print and thirty (30) from radio media. The decision variables are integer variables defined as:
X_{ti} : Number of TV media ($i = 1, 2, \ldots 5$)(t-index for TV)
X_{pj} : Number of Print media ($j = 1, 2, \ldots 10$)(p-index for Print)
X_{rk} : Number of Radio media ($k = 1, 2, \ldots 30$)(r-index for Radio)

Table 2. Cost of advert and estimated audience exposure for TV media

Television media	Cost (Gh¢) per advert	Estimated Audience Exposure
Meto TV	780	447,599
GTV	900	895,199
Viasat 1	500	413,169
TV3	427	516,461
Crystal TV	590	103,292

Table 3. Cost of advert and estimated audience exposure for radio media

Radio media	Cost (Gh¢) per advert	Estimated Audience Exposure	Radio media	Cost (Gh¢) per advert	Estimated Audience Exposure
Adom FM	103	65,655	Kapital Radio	90	11, 197
Hitz FM	68	32, 827	Nhyira FM	25	78,381
Joy FM	151	87,541	Otec FM	42	55,986
Happy FM	25.9	32,827	Kessben FM	50	45,565
Asempa FM	42	21,885	Radio Capital	25	25,888
Radio Gold	35.7	32,827	Sunrise FM	98	21,885
Citi FM	45	21,885	Kyzz FM	40	31,074
Unique FM	62	3,884	Aseda FM	35	3,305
BBC Radio	87	3,884	Melody FM	25	23,305
Choice FM	76.8	10,942	Volta Star	95	10,942
Hot FM	45	32,827	Savana Radio	25	21,885
Obonu FM	48.7	10942	Radio UW	69	16,798
Hello FM	83.9	34,368	Radio Bar	45	98,651
Love FM	64	67,184	Peace FM	42	28,279
Angel FM	48.7	3,884	Uria Radio	26	20,597

4.2 The Objective Function

The objective of the problem is to allocate the advertising budget across the various media so as to maximize the total target audience exposure for the campaign. Buying a unit space or time in each media is expected to lead to a certain number of audience exposures. Therefore, audience exposure associated with each media is a parameter of the objective function. Supposing that $A = (A_{ti}, A_{pj}, A_{rk})$ and $X = (X_{ti}, X_{pj}, X_{rk})$ where A_{ti}, A_{pj}, A_{rk} are respectively row vectors for TV, Print, and Radio audience exposures, X_{ti}, X_{pj}, X_{rk} representing the decision variables, then the objective function is formulated as follows;

$$Maximize \qquad Z = AX \tag{4.1}$$

where

$$AX = \sum_{i=1}^{5} A_{ti}X_{ti} + \sum_{j=1}^{10} A_{pj}X_{pj} + \sum_{i=1}^{30} A_{rk}X_{rk} \tag{4.2}$$

4.3 The Budget constraints

Let (C_{ti}, C_{pj}, C_{rk}) be row vectors for TV, Print, and Radio cost of advert respectively. Let (X_{ti}, X_{pj}, X_{rk}) represent the decision variables of the problem whilst maintaining the definition of the other indices; the total budget constraint is developed in a similar manner as in section 4.2 above.

- Total budget for TV media

$$\sum_{i=1}^{5} C_{ti}X_{ti} \le 200,000 \tag{4.3}$$

- Total budget for Print media

$$\sum_{j=1}^{10} C_{pj}X_{pj} \le 150,000 \tag{4.4}$$

- Total budget for Radio media

$$\sum_{i=1}^{30} C_{rk}X_{rk} \le 250,000 \tag{4.5}$$

4.4 Media Advert Constraints

- TV Media: Maximum advert for each TV media.

$$X_{ti} \le 270 \qquad \forall i \tag{4.6}$$

- Print Media: Maximum advert for each Print media.

$$X_{pj} \le \{90, 12, 36, 24, 12, 12, 90, 12, 90, 36\} \qquad \forall j \tag{4.7}$$

- Radio Media: Maximum advert for each Radio media.

$$X_{rk} \le 270 \qquad \forall k \tag{4.8}$$

The complete LP model resulting from equations 4.1 to 4.8, together with the non-negativity constraint, has 45 decision variables and 48 constraints expressed as:

$$\left.\begin{array}{ll}
\text{Maximize} & Z = \sum_{i=1}^{5} A_{ti}X_{ti} + \sum_{j=1}^{10} A_{pj}X_{pj} + \sum_{i=1}^{30} A_{rk}X_{rk} \\
\text{Subject to} & \sum_{i=1}^{5} C_{ti}X_{ti} \le 200,000 \\
& \sum_{j=1}^{10} C_{pj}X_{pj} \le 150,000 \\
& \sum_{i=1}^{30} C_{rk}X_{rk} \le 250,000 \\
& X_{ti} \le 270 \qquad \forall i \\
& X_{pj} \le \{90, 12, 36, 24, 12, 12, 90, 12, 90, 36\} \qquad \forall j \\
& X_{rk} \le 270 \qquad \forall k \\
& X_{ti},\ X_{pj},\ X_{rk} \ge 0
\end{array}\right\} \tag{4.9}$$

5 Results and Discussion

The output of the model is presented in Table 4 which gives the recommended media mix for the study. These include three (3) Television media outlets, two (2) print and ten (10) Radio media outlets. These recommended media mix identified generates an optimal value of 635,048,700 target audience exposure.

Table 4. Results of the LP

Variable and Media Outlets	No. of adverts	Variable and Media Outlets	No. of adverts
TELEVISION			
X_{t1}: Metro TV	0	X_{r24}: BBC Radio	0
X_{t2}: GTV	270	X_{r25}: Choice FM	0
X_{t3}: Viasat 1	199	X_{r26}: Hot FM	0
X_{t4}: TV3	270	X_{r27}: Obonu	0
X_{t5}: Crystal TV	0	X_{r28}: Angel FM	0
		X_{r29}: Hello FM	0
PRINT		X_{r30}: Love FM	270
X_{p6}: Daily Graphic	0	X_{r31}: Kapital Radio	0
X_{p7}: Graphic Showbiz	0	X_{r32}: Nhyira FM	270
X_{p8}: Ghanaian Business	0	X_{r33}: Otec FM	270
X_{p9}: Junior Graphic	24	X_{r34}: Kessben FM	270
X_{p10}: Ghanaian Sports	0	X_{r35}: Radio Capital	270
X_{p11}: The Mirror	0	X_{r36}: Sunrise FM	0
X_{p12}: Daily Guide	0	X_{r37}: Kyzz FM	0
X_{P13}: The Chronicle	0	X_{r38}: Aseda FM	0
X_{p14}: Ghanaian Times	0	X_{r39}: Melody	270
X_{p15}: Ninety Minutes	36	X_{r40}: Volta Star	0
		X_{r41}: Savana Radio	270
RADIO		X_{r42}: Uria Radio	0
X_{r16}: Adom FM	0	X_{r43}: Radio UW	0
X_{r17}: Hitz FM	0	X_{r44}: Radio Bar	270
X_{r18}: Joy FM	0	X_{r45}: Peace FM	0
X_{r19}: Happy FM	270		
X_{r20}: Asempa FM	0		
X_{r21}: Radio Gold	270		
X_{r22}: Citi FM	0		
X_{r23}: Unique FM	0		

OPTIMAL VALUES: 635, 048,700

The output shows that advertisement should be made 270 times in GTV, 199 times in Viasat 1 and 270 times in TV3 was enough to yield the overall exposure for the television media categories. For the print (Newspapers), advertisement should be made 24 times in Junior Graphic and 36 times in Ninety Minutes as indicated in Table 4.

For the radio stations, advertisements should be made 270 times in each of Happy FM, Radio Gold, Love FM, Nhyira FM, Otec FM, Kessben FM, Radio Capital, Melody FM, Savanna Radio and Radio BAR, as indicated by the basic variables in the solution given above. The variables such as Metro TV, Crystal TV, Daily Graphic and Peace FM, having solution of zero at optimum level are non-economical to use. Though estimated target exposure values in some of them may be encouraging such as Metro TV, Daily Graphic and Citi FM, they are non-optimal decision variables.

Finally, the slack or surplus solutions at optimality informed the decision maker that either there is more of advertising space or time in a particular media or the advertiser has to secure extra advertising space or time in some of the media to run the advertising campaign at the expense.

Sensitivity analysis test was conducted on some of the parameter values and the resulting models were compared with the original LP problem. To carry out the sensitivity analysis, the right-hand-side (RHS) of the budgetary constraints values was varied at 10% level, so as to gauge the effect on the objective function value. Each of the right hand side (RHS) of the budgetary constraints values was decreased and increased 10% which respectively led to a corresponding decrease and increase in the optimal value to 585, 346,000 and 683, 098, 600 exposures per the length of the program. When the RHS budgetary values were varied, it was observed that new variables either enter or leave the basis which led to the creation of new media mix for the advertising campaign program. Again, some of the basic variables increased or decreased in solution values leading to a subsequent effect on the optimum value.

Similarly, each of the media category exposure values were increased and decreased one at a time by 10% which led to a corresponding increase and decrease in the objective function value recording 698, 553,558 and 571,543,837. Finally, a sensitivity analysis on all the audience exposure and the RHS budgetary constraints values was carried out by varying them up and down by 10%. A summary of the post-optimality test carried out in this study is shown in Table 5.

Table 5. Summary of sensitivity analysis

Media category and budget	Optimal value	Optimal Value up by 10%	Optimal Value down by 10%
TV Media Group	635, 048,700	660,108,455	609,988,945
Difference	0	25,059,755	25,059,755
% value	0	4%	4%
Print Media	635,048,700	647,359,074	622,738,326
Difference	0	26,134,740	26,134,740
% value	0	4%	4%
Audience Exposure values	635,048,700	698,553,558	571,543,837
Difference	0	63,504,858	3,504,863
% value	0	10%	10%
Budget values	635,048,700	683,098,600	585,346,000
Difference	0	48,049,900	49,702,700
% value	0	8%	8%

All these variations in the model notwithstanding, some of the media outlets such as GTV, viasat 1 and TV3 for television media, Junior Graphic and Ninety Minutes for the print and Happy FM, Radio Gold, Love FM, Nhyira FM, Otec FM, Kessben FM, Radio Capital, Melody FM, Savanna Radio and Radio BAR for Radio media group always remain in the basis regardless of any variation in the model.

6 Conclusion

We have shown that out of the three major media categories which give a total of Forty-Five (45) media outlets in the country and advertising budget of Six Hundred Thousand Ghana Cedis (GH600,000), the optimal target audience exposure is 635,048,700. The optimum media mix generating this objective value include; three (3) Television, two (2) newspapers, and ten (10) FM stations, which is achieved using our method. The sensitivity analysis carried out shows that some media outlets such as Junior Graphic, and Ninety minutes, Radio Gold, Happy FM, Love

FM, Radio BAR etc., and GTV, Viasat 1 and TV3 had a tremendous impact on the optimal value since they always remain in the basis regardless of any variation in the model.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Cosgun O, Gultas I, Serarslan MN. Application of a mathematical model to an advertisement research problem. An International Journal of Optimization and Control: Theories & Applications. 2013;3(1):23-33.

[2] Courtney Paulson, Lan Luo, Gareth M. James. Efficient large-scale internet media selection optimization for online display advertising. Journal of Marketing Research. 2018;55(4):489-506.

[3] Hooshang Tadrisi Javan, Amir Khanlari, Omid Motamedi, Hadi Mokhtari. A hybrid advertising media selection model using AHP and fuzzy-based GA decision making. Neural Computing and Applications. 2018;29(4):1153-1167.

[4] Broadbent SR, Beale EML, Hughes PAB. A computer assessment of media schedules. Operational Research Quarterly. 1966;17:381-412.

[5] Kotler, Philip. Marketing decision making: A model building approach. New York, Galt, Rinehart and Winston; 1971.

[6] Day RL. Linear programming in media selection. Journal of Advertising Research. 1962;2(2):40-44.

[7] Kwansah-Aido K. Events That Matter: Specific Incidents, Media Cover-age, and Agenda-Setting in a Ghanaian Context. Canadian Journal of Communication. 2003;28 (1):43-66.

[8] Brown DB, Warshaw MR. Media selection by linear programming. Journal of Marketing Research. 1965;2(1):83-88.

[9] Stefanos K, Evangelos CT, Theodoros P. Allocating advertising expenses using linear programming and MS-Excel. Proceedings of the 16th national conference of the Hellenic or society: Project management and administration. 2003;139-144. P.G. Ipsilantis ed.

[10] Bass FM, Lonsdale RT. An exploration of linear programming in media selection. Journal of Marketing Research. 1966;3:79-88.

[11] Chang W, Ng M, Yuen W, Zhang S. A linear programming approach for determing optimal advertising policy. IMA Journal of Management Mathematics. 2006;17:83-96.

[12] Twum SB, Aspinwall E, Fliege J. A multicriteria optimization model for reliability design of series-parallel systems-Part I. International Journal of Quality & Reliability Management; 2011.

[13] Murtagh BA, Saunders MA. Large-scale linearly constrained optimization. Mathematical Programming. 1978;14:41-72.

[14] David R. Anderson, Dennis J. Sweeney, Thomas A. Williams, Jeffrey D. Camm, James J. Cochran, Michael J. Fry, Jeffrey W. Ohlmann. An introduction to management science: Quantitative approaches to decision making. Fifteenth Edition; 2017.

[15] Sharma JK. (2013). Operations research; theory and application. Trinity Press, Macmillan Publishers India Ltd., 5th Edition.

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