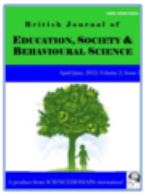

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Activities for Math Circles

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Abstract

In this paper, we describe various mathematical activities designed for and used in the Math Circles in North Bay, Ontario. These activities were designed to accommodate the particular format of Math Circles, the age group of the participants, the different stages of the lesson in which they were used and other educational goals. We discuss the appropriateness of these activities and their contribution towards the overall goals of the program.

Keywords: Mathematical circles, mathematical competitions, gifted children, learning activities

1 Introduction

Math Circles in North Bay is dedicated to providing school-age children in the North Bay area with the opportunity to explore mathematics outside the prescribed school setting and curriculum [1]. Math

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Circles also strives to create a rich learning environment where school-age children interested in mathematics can share their ideas, make lasting relationships with like-minded children and reach their full potential. The program also serves as a preparation for participation in the annual Math Kangaroo contest, held annually in late March, [2]. The program is hosted by the Department of Computer Science and Mathematics at Nipissing University and co-sponsored by NUMERIC [3]. In a broader context, Math Circles is an enrichment activity that exists in various forms throughout the world. Experiences of running such programs at local and national level are well documented in the literature: [4,5]. Additionally, problems sets, topics and even day-by-day pedagogical materials have been published: [6,10]. Often such programs feature substantial historical content [11-13], which is usually a great motivational tool. In most of these places, the local context is one of mathematically rich environment, where on top of an excellent general educational system, a system for identifying the talented children and working with them is in place. The local context in North Bay is quite different - a small Northern community where no significant resources, material or human are available to support the identification and special attention to advanced learners in mathematics (and in other subjects). With these peculiarities and special educational goals in mind, the following are some activities used at Math Circles in North Bay to create varied and engaging learning opportunities.

2 Activities

2.1 Activity 1: The Fly Swatter Game

Materials:
2 fly swatters
Flat surface that can be written on (i.e. white board, chalk board, Smart board etc.)
Flash cards (optional)

Activity:
Hands down, the biggest advantage to this activity is its versatility. The Fly Swatter Game could be used in conjunction with almost any topic, math related or otherwise. It is very simple, easy to learn and to teach and it is a hit with our younger and older participants alike.

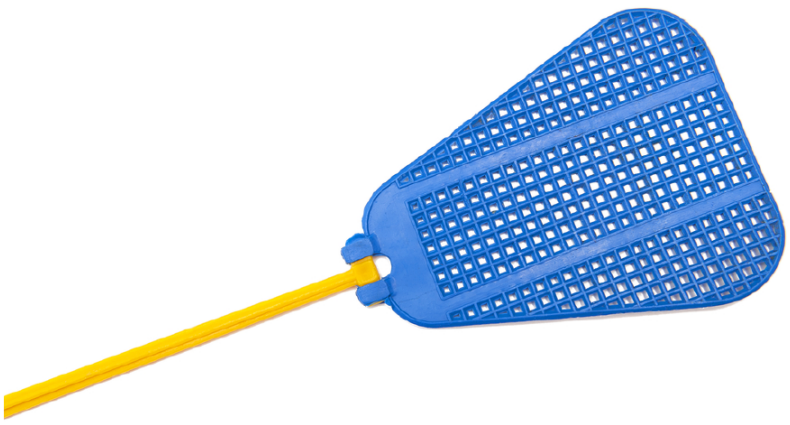

Figure 1: The static case.

Figure 2: Flash cards for the fly swatter game

The basic idea of the game is a race. Two participants face off against one another. The white board, chalk board, SMART board etc. is covered in possible answers to a certain type of question. The game facilitator then asks a question for which the answer is located somewhere on the board. The first person to hit the correct answer wins the round. If someone hits the wrong answer they are disqualified and the other person then can guess with no pressure. Sometimes there is not a winner if both participants choose incorrectly. Ties are usually easily settled either because one fly swatter is beneath another or by a quick rematch. The winner gets to challenge the next participant while the loser is finished playing until their next turn.

For example, a popular topic we use at Math Circles in North Bay (especially for the first few sessions) is arithmetic. A variety of numbers are written at random all over the given board. Then the game facilitator either uses flash cards to tell the two contestants the question or makes it up in their head and calls it out to them. Using the topic of arithmetic makes it very easy to adjust the difficulty of the question being asked to all levels of participants present. For example, a student in grade 3 might be asked, "what is $(3 \times 5) - 3$?", while someone in grade 8 or 9 might be asked, "what is 84 divided by 12 and then plus 10 and then multiplied by 4?". Arithmetic is a good topic to review often, especially for the junior elementary grades (4-6), so that the participants do not get caught up in basic arithmetic while attempting to apply their newly developed problem solving skills.

If the topic of discussion for the session is geometry, a variety of shapes could be placed randomly on the board, perhaps even with dimensions, and questions asked about the name of the shape and its perimeter, area and volume where applicable. The participants can be divided into teams based on age, grade, gender, hair colour etc. and they can even compete against student volunteers or faculty members. Students can gain points and the activity can even become an ongoing competition between pre-set teams if the attendance of participants is regular or between the participants and student volunteers and faculty.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3: The playing board

2.2 Activity 2: Math Check

Materials:
Math Check game board
Math Check markers (1–10/11)

Activity:
Math Check is another very popular activity at Math Circles meetings in North Bay. The concept of the game is loosely based on Checkers. The Math Check board is rectangular divided into a total of 60 smaller rectangles. Each rectangle is numbered from 1 to 60 with the marker 10 being labeled 10/11.The markers match the first ten small rectangles. They are small rectangles with the numbers 1 to 10/11 on them.

The object of the game is to eliminate all of your opponents markers. To begin the game, the first player chooses to put as many of their markers onto the first row of the board as they like. However, a marker may only be placed on a rectangle of the board if the number on the marker is a factor of the number on that rectangle of the board. For example, if player one wished to place their marker 2 onto the board, they could place it anywhere in the first row of the board on their side on a number that is a multiple of 2 (i.e. 2, 4, 6, 8, or 10/11). Similarly, player 2 then can choose to place as many of their markers as they wish onto their side of the board. Again, they may only place a marker on a rectangle of the board if the number on the marker is a factor of the number on that rectangle of the board. In this way, player 1 may be able to place their markers with less difficulty. If the players are young and/or have limited quick recall of multiplication facts; therefore, we try to pair students accordingly when they play this game.
To continue, once both players have placed their markers onto the game board, player 1 then can choose to either add more markers to the board (if they did not already add all their markers) or to

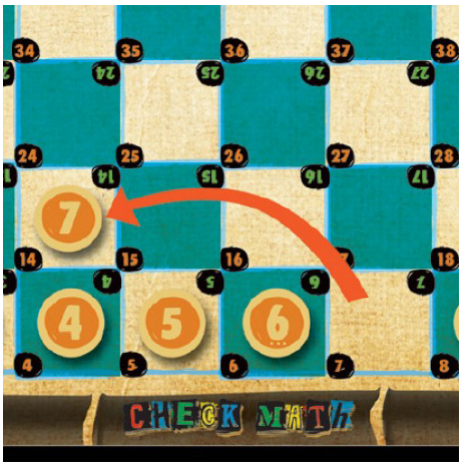

Figure 4: Part of the Check Math board with a possible move

move a marker forward. To move a marker to the next row is very simple. The player can move the marker to any rectangle in the next row that is a multiple of the number on the marker. A player can only move one marker forward each turn and can only move the marker forward one row each turn. A player may not move a marker backward a row or sideways along a row. To eliminate another player's marker one must land on the same square as their opponent's marker. Players must move a marker or add a marker to the board at each turn (they cannot pass on their turn).
If a player's marker makes it to the opposite side of the game board, that marker can then move back toward them but it can no longer move forward (i.e. a marker can never move both forward and backward). The game ends when a player eliminates all of their opponents markers or there are no more moves available for either player to make. If the game ends because of a stalemate, the player with the most markers left on the board wins.
The educational value of this activity goes beyond simple strengthening of multiplication and division skills. It involves designing and implementing strategies, based on the opponents possible moves, possible responses to these moves, etc. Then, it involves re-evaluation and modification of these strategies, based on the actual moves of the opponent.

2.3 Activity 3: The Dice Game

Materials:
6 dice
Paper and pencil (optional)

Activity:
The Dice Game is a very simple and yet very useful and potentially challenging game. It is a game that can be played alone or with a group of people. To begin the game, the player rolls 2 of the 6 dice and finds the sum of the two dice rolled. Then, the other 4 dice are rolled and the player must find a way to combine all 4 numbers using addition, subtraction, multiplication and/or division to create the sum rolled with the first 2 dice. Each of the 4 numbers rolled can only be used once. More dice can be added to make the game more challenging or it can become a competition between players or teams to see who can come up with a solution first.

Figure 5: Sample roll of a set of 6 dice

In the example shown in Figure 5, the 6 dice rolled had 5, 4, 6, 1, 2, 4 on their upper faces. Thus, we will have $5+4=9$ and the players have to make up 9 using the numbers 6, 1, 2, and 4. One possible solution is to have $6+1-2+4=9$.
The Dice Game is highly useful in helping participants to hone their mental math skills as it requires players to have strong and quick mental computational knowledge to be successful. Students must quickly compute sums, differences, products and divisions in their minds (or rough them out on paper) and rapidly consider and then reject or accept possible combinations of numbers and expressions. Students can also develop their own strategies for success at the game and possibly even share those strategies with one another to compare different avenues of thought. For example, most students quickly realize that 1 is an easy number to use because multiplying by 1 will not change the answer and therefore, the 1 does not necessarily need to be considered.

2.4 Activity 4: Tangrams

Materials:
Standard tangram set(s)
Sheet(s) with shapes to be made

Activity:

Tangrams is another great activity to engage students in mathematical thinking while also having fun and getting to know one another. Students are given a variety of different size triangle and rectangle cut-outs of paper. They are then asked to use those shapes to create one overall shape. This activity is very valuable in helping students to develop strong visual-spacial skills and problem solving strategies. Students must visually breakdown the overall shape into its smaller components which can be very challenging even for the most experienced players. Students need to develop strategies such as deciding how to prioritize where to place each shape. For example, sometimes in the overall shape it is clear that a large square cut-out has limited placement options.

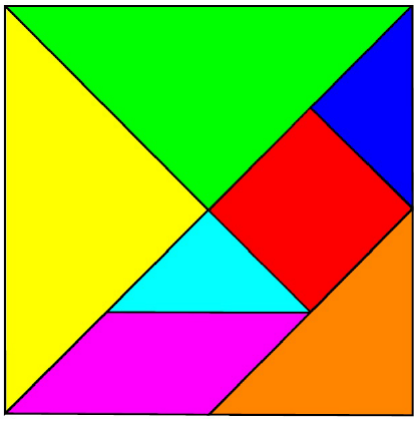

Figure 6: Standard Tangram set

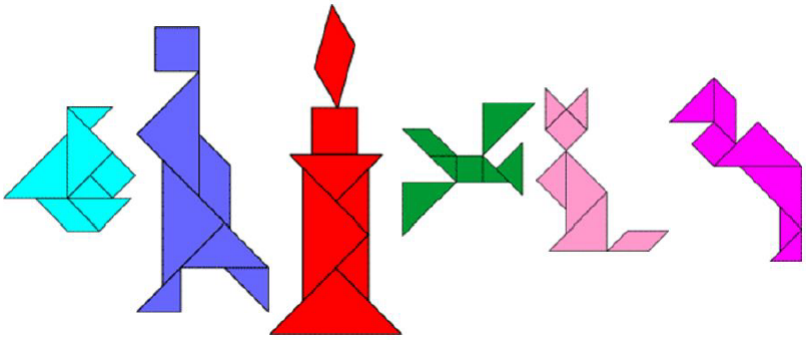

Figure 7: Tangram shapes

3 Evaluation and Discussion

The activities presented in this paper were used in conjunction with a multitude of other activities and formats of Math Circles in North Bay. For a detailed description of the scope and the format of these, please refer to the companion paper, [1]. The evaluation of the activities and their contribution to the educational goals of the program is done internally among the members of the team: students who design and lead the activities and the faculty members who supervise the program. In addition to their opinions and points of view, a large volume of informal input from the participants, and sometimes from their parents is taken into consideration. It was not an intention of this paper to include evaluation of these, or any other activities through questionnaires or any other formal type of input from the participants. The main reason for this is that due to their age group (often grades 3 to 6) this will results in unreliable responses or biased ones. In general, the participants were very enthusiastic about most of the activities included in the Math Circles, and to the entire program. This is demonstrated by the growth of both the Math Circles program throughout the last four years in terms of participants and the Math Kangaroo participation of the students from Math Circles. These are discussed, once

again in [1]. We would like to add that further justification of the appropriateness of the activities to the educational goals of the Math Circles and of their success are the following two facts: the retention rate of the Math Circles program is 100%, all the children who participated in this program previously continue to participate in it; the results of these students in Math Kangaroo are improving steadily from year to year, and at the North Bay regional contest centre as a whole. Further, the growth in the numbers of participants in the last two years is clearly attributable (through parent's testimonies) to children including their friends in it, recommending the program, and to teachers or parents performing these activities in their classrooms or at home. Finally, the activities presented in this paper are only a representative sample, and the most original, interesting and successful ones. We encourage the discussion with colleagues and other members of the community that are interested in similar types of educational experiences, this is one of the major goals of the paper.

4 Conclusions and Future Work

The educational activities presented in this paper are developed by the student volunteers in Math Circles in North Bay, Ontario or they are adaptations of well-known math games and puzzles. These activities help engage school-age children in problem solving activities and develops their mathematical thinking, promoting interest in and involvement with mathematics at the same time. The development and adaptation of learning activities also serves an important role in the process of education of future mathematical teachers - career chosen by many of the students who lead Math Circles.

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Competing Interests

The authors declare that no competing interests exist.

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