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The Use of Mathematical Model to Predict Levels of N and K for Optimum Yield

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

Sometimes, yield from a commercial plantation is reduced because of inadequate availability of fertilisers to the plant due to various reasons such as poor quality of fertilizers used, loss of fertilizers in the field during cultivation due to natural causes etc. One way of preventing such loss in yield is to apply remedial fertiliser doses in the field in an intermediate stage of cultivation. To implement such a method effectively, a mathematical model has been proposed in this work.

The model first determines the amount of fertilisers needed at the beginning of cultivation to optimize yield. It then ascertains at an intermediate stage of cultivation whether the plant receives adequate fertilisers for producing optimum yield. In case it is found out that required fertilisers are not available to the plant, the model decides how much more remedial fertilizer doses should be applied at the intermediate stage so that yield will not be affected. A potato plantation has been considered to illustrate the applicability of the proposed mathematical model.

Keywords: Fertilizer response; regression models; potato plant; fertilizer nitrogen; fertilizer potassium; newton's method; mathematical optimization; marquardt's method.

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1. INTRODUCTION

Due to several factors, sufficient nutrients may not be available to the plant to enable it to produce maximum yield. For instance, applied fertilisers in the soil can be washed away from farm fields into water ways during excess rain. Nitrogen can be lost from farm fields in the form of gaseous nitrogen based compounds like ammonia and nitrogen oxides. There are several instances where cultivation suffers due to low quality of fertilisers used. The amount of fertilisers purchased by the farmers and applied in the field depends on weights and measurements which may not be perfect. Thus, a farmer may have the notion that the required amount of fertilisers have been used in the field, but in reality, the crop may not collect adequate fertilisers from soil in order to produce anticipated yield.

The mathematical model suggested by us can remedy the above mentioned problems to a great extent. The model uses the data generated from experimental results in a trial plantation in the vicinity of a commercial plantation. The model has the following advantages

(i) It determines the amount of fertilisers that need to be applied in the field at the beginning of the cultivation in order to produce optimum yield.
(ii) It determines the amount of nutrient uptake required by the plant at an intermediate stage of cultivation for producing optimum yield.
(iii) It detects deficit in availability of fertilisers to the plant in the intermediate stage of cultivation.
(iv) If there is a deficit, the model determines the additional remedial fertiliser doses to be applied in the field at the intermediate stage of cultivation so that yield is not affected adversely.

We have illustrated the applicability of the mathematical model by considering experimental data of potato [1]. Here data from 4^2 factorial fertiliser rate trial with four rates of N and four rates K and a total of 16 treatments (i.e. all combinations of four rates of nitrogen and four rates of potassium) have been taken into account to develop functions representing the response surfaces of yield and N and K plant uptakes. These response surfaces have been used in the model. The response surfaces have been derived by applying multiple linear regression [2].

Several experiments have been undertaken in the past to study the effect of N and K fertilizers on yield of potato. The supply of NPK fertilizers to optimize yield of potato has been studied by Kolbe et al. [3]. In their work, the objective function representing the yield, subject to some linear constraints, was optimized by an unconstrained sequential optimization technique. In the work of Koech et al. [4], the 3D response surfaces were employed to evaluate the effect of NPK fertilizers on yield of potato by varying two variables within the experimental range under investigation and holding the other variable at its central level. *Piekutowska* et al. [5] developed a linear model based on multiple linear regression analysis and a nonlinear model based on artificial neural network to predict potato cultivation before harvest. In most of the works in the past, the yield and nutrient uptake response surfaces have been assumed to be quadratic or linear. It is easier to optimize a quadratic function of several variables. However, it is not true that the response surfaces will be quadratic functions always, since such surfaces may not fit the data well. Here we have allowed the response surfaces to be of higher degree in order that they can be statistically significant. If the response surfaces are not quadratic, the mathematical techniques used in earlier works to maximize function of several variables are not applicable and we need to apply more advanced techniques.

The experimental data required by the mathematical model have been presented in section-2. In section-3, a step by step computational procedure of the proposed mathematical framework has been presented. In section-4, applicability of the mathematical model has been illustrated taking into account the data on potato plantation presented in the Section-2.

2. MATERIALS AND METHODS

2.1 Description of Trial

The current study is developed using the data generated from an investigation on potato plantation under field condition during January to April [1] at the Horticultural research station, Gandhi Krishi Vignana Kendra, University of Agricultural sciences, Bangalore, India. It is geographically situated at $77^0 35'$ East longitude and $12^0 58'$north latitude at an altitude of 930 meters above the sea level. The soil of the experimental area was sandy loam.

The land was ploughed once, followed by digging with hand guddali and was levelled. The field was divided into experimental units of 4.8m ×2.4m with irrigation channels in between two experimental units. The trial used a randomized complete block design of two replicates with16 treatments .The 16 treatments are the factorial combination of 4 rates of nitrogen $(0,80,160\ and\ 240\ kg/ha)$ and four rates of potassium $(0,60,120 and\ 180\ kg/ha)$. Certified seed of tubers of cultivator Kufri Jyothi weighing 35-50 kg of some physiological age were selected. They were soaked in Diethane M-45 (0.25%) for 10 minutes, dried in shade and were used for planting.

The nitrogen in the form of calcium ammonium nitrate containing 20%N and potassium in the form of mutilate of potash containing 60%K were used in the trial. The experimental units were given light irrigation for two days prior to plantings and were dug after two days when the soil was optimum .About 5-6 cm deep furrows were made at a distance of 60 cm and the treated tubers were planted in the furrows at a distance of 20 cm $(intra\ spacing\ )$ in such a way that planted tubers were 3-4 away from the fertilizers.

Nitrogen in plant samples was estimated by Microkjeldahl method [6]. Potassium content in plant samples was determined by flame photometer using specific filter and LPG flame.

2.2 Statistical Analysis

For our mathematical framework, we need three response surfaces depicting the effect of different levels of N and K fertilisers in trial plantation. They are

(i) The yield response surface of potato due to application of N and K fertilisers at the beginning of cultivation
(ii) Response surfaces of plant N and K uptake at the intermediate stage of cultivation, expressed as functions of applied N and K fertilisers at the beginning of cultivation.

Instead of studying effect of N and K fertilisers separately on yield, the combined effect of both the fertilisers on the yield has been considered because significant correlation between levels of N and K and yield has been reported by Singh et al. [7], Ali et al. [8].

The effect of different level of N and K on yield of potato is provided in Table 1.The response surface for yield, after application of multiple linear regressions [2] using data in Table 1, is given by

$$y(N,K) = a + b\ N + c\ N^2K + d\ K^2N + f\ N^3 + g N^2K^2, \quad (2.1)$$

where $y(N,K)$ is the yield of potato, a, b, c, d, f and g are regression coefficients having values

a= $109.1525\ (P < .001)$, b=0.710455 $(P = .001)$, c=2.63e-05 $(P = .01)$, d=3.2e-05 $(P = .02)$, f=$-6.2e - 06(P = .07)$, g=-3e-07 $(P = .001)$ .The yield response surface has coefficient of determination $(R^2 = 0.89)$ and $(adjusted R^2 = 0.84$ with significance F $< .001$.

The effect of different level of N and K on plant uptake of N in the trial potato plantation is provided in Table 2.The response surface for uptake of nitrogen, after application of multiple linear regressions [2] using data in Table 2, is given by

$$[N] = y_N(N,K) = a + bN + cK + dN^2 + eNK + fK^2 + gN^3, \quad (2.2)$$

where $y_N(N,K)$ represents uptake of N by plant and a ,b, c, d, e ,f and g are regression coefficients having values a=0, b=1.867594$(P = .03$, c=1.764794$P<.001$, d=-0.00922$P=.30$, e=-0.00336 $(P = .01)$, f=-0.00438 $(P = .07)$,g=1.9e-05 $(P = .40)$ .The nitrogen uptake response surface $y_N(N,K)$ has coefficient of determination (R^2) of 0.98 and Adjusted (R^2) of 0.86 with significance F< .001.

The response surface for uptake of potassium, after application of multiple linear regressions [2] using data in Table 3, is given by

$$[K] = y_K(N,K) = a + bN + cK + +dNK + eK^2, \quad (2.3)$$

where $y_K(N,K)$ represents uptake of K by plant and a ,b ,c ,d and e are regression coefficients having values : a=114.8226 $(P < .001)$, b =0.227148 $(P < .001)$, c=0.679077 $(P < .001)$, d=0.000665 $(P = .10)$, e=-0.00259 $(P < .001$.The potassium uptake response surface yKN,Khas coefficient of determination$R2=.96$ and $(adjusted\ R^2 = .94)$ with significance F< .001.

Table 1. Effects of different level of N and K on yield of tubers (kg/ha) in factorial fertilization trial plantation of potato

Nitrogen levels(N)(kg/ha)	Potassium levels(K)(kg/ha)	Yield(Y) (kg/ha)
0	0	94.41
0	60	114.55
0	120	117.19
0	180	126.43
80	0	170.49
80	60	148.61
80	120	203.54
80	180	180.7
160	0	185.97
160	60	250.31
160	120	228.27
160	180	266.74
240	0	199.06
240	60	244.41
240	120	237.29
240	180	139.69

Table 2. Total plant uptake of N by different levels of N and K)(Kg/ha) in factorial fertilization trial plantation of potato)

Nitrogen levels(N) (kg/ha)	Potassium levels(K) (kg/ha)	plant uptake of N (kg/ha)
0	0	129.78
0	60	150.71
0	120	163.51
0	180	175.35
80	0	150.88
80	60	169.3
80	120	196.46
80	180	202.05
160	0	158.27
160	60	192.99
160	120	205.04
160	180	227.96
240	0	176.83
240	60	201.05
240	120	228.27
240	180	238.72

2.3 Methodology

The execution of the model starts with the calculation of the parameter values of N and K fertilizers that need to be applied at the beginning of cultivation to maximize yield .To this end, the yield response surface equation (2.1) is maximized by an unconstrained nonlinear optimization technique. There are several nonlinear optimization techniques [9]. Marquardt's Method [9] is preferred here because of its wider scope of applicability. To address crop productivity through soil test based plant nutrient management, the ICAR project on *Soil Test Crop Response* has used a targeted yield approach to develop relationship between crop yields on the one hand, and soil test estimates and fertilizer inputs, on the other. In this targeted yield approach, it is assumed that there is a linear relationship between grain yield and nutrient uptake by the crop and for obtaining a targeted yield. A linear relationship may not exist between yield and nutrient uptake. Marquardt's Method is capable of handling implicit nonlinearity between grain yield and nutrient uptake.

Table 3. Total plant uptake of K by different levels of N and K) (Kg/ha) in factorial fertilization trial plantation of potato

Nitrogen levels(N)(kg/ha)	Potassium level (K)(kg/ha)	plant uptake of K (kg/ha)
0	0	126.86
0	60	146.67
0	120	158.59
0	180	157.03
80	0	139.2
80	60	164.43
80	120	167.34
80	180	188.04
160	0	148.86
160	60	193.79
160	120	211.77
160	180	202.26
240	0	164.13
240	60	212.15
240	120	241.63
240	180	232.73

The parameter values of N and K fertilizer that optimize yield are substituted in uptake response surfaces (2.2) and (2.3). The value of N and K, thus obtained, are the uptake level of fertilizers a plant is expected to have at the chosen intermediate stage of cultivation in order that it can produce optimum yield. Let the

The plant nutrient uptakes of commercial plantation in the intermediate stage of cultivation are collected and are compared with corresponding N and K values necessary for optimum yield found out in trial plantation. If it is found that the uptake level of commercial plantation is less than the uptake levels required for optimizing yield, it is concluded that there is a deficit in fertilizers available to the plant and additional doses of fertilizers are required in commercial plantation at the intermediate stage.

To find out the additional doses of fertilisers, the model first finds out the amount of fertilisers available to the commercial plantation till the time when plant uptake levels were measured. A solution of a system of two nonlinear equations, as mentioned step-6 bellow, is required to find out the fertilizer available to commercial plantation. We do not have any closed form formula to solve a system of equations in which member equations have degree more than two. We have to apply a numerical method to solve the system. Since the equations in our mathematical model may have degree more than two, a numeric technique, namely Newton's Method [10] is applied by us. The differences between the fertilizers that should have been used for optimal yield and the fertilizers used in commercial plantation in the past, are the corrective fertilizer doses that need to be prescribed at the intermediate stage of cultivation to commercial plantation.

We summarize the step by step procedure of the mathematical model adopted by us bellow.

Step-1: Find the three response surfaces (N, K), $y_N(N, K)$, $y_K(N, K)$ by multiple linear regressions.
Step-2: Solve the following unconstrained optimization problem Maximize $y(N, K)$.

by Marquardt's Method. Let the value of N and K which optimizes $y(N, K)$ be denoted as N_{opt} and K_{opt} respectively.

Step-3: Compute $y_N(N_{opt}, K_{opt})$ and $y_K(N_{opt}, K_{opt})$from equations (2.2) and (2.3). Let $[N]_{opt} = y_N(N_{opt}, K_{opt})$ and [K]opt $= y_K(N_{opt}, K_{opt})$.
Step-4: Let $[N]_{app}$ and $[K]_{app}$be the Nitrogen uptake and Potassium uptake respectively in commercial plantation at the intermediate stage of cultivation.
Step-5: (i) If $[N]_{app} \geq [N]_{opt}$ and $[K]_{app} \geq [K]_{opt}$, we conclude that the fertilizers used in the commercial plantation are sufficient to produce optimum yield. Go to step-8 (ii) If$[N]_{app} < [N]_{opt}$ or$[K]_{app} < [K]_{opt}$, go to step 6

Step-6: Solve the following system of nonlinear equations by Newton's Method [10]

$y_N(N, K) = [N]_{app}$,

$y_K(N, K) = [K]_{app}$.

Let the solution of the system of equations be Napp and Kapp

Step-7: The additional amount of N and K fertilizer required to be applied to the commercial plantation are N_{opt} - N app and K_{opt} - Kapp depending on whether $[N]_{app} < [N]_{opt}$ or $[K]_{app} < [K]_{opt}$.

Step-8 Stop.

The algorithm of computational steps is also presented in form of flow chart in Fig. 1.

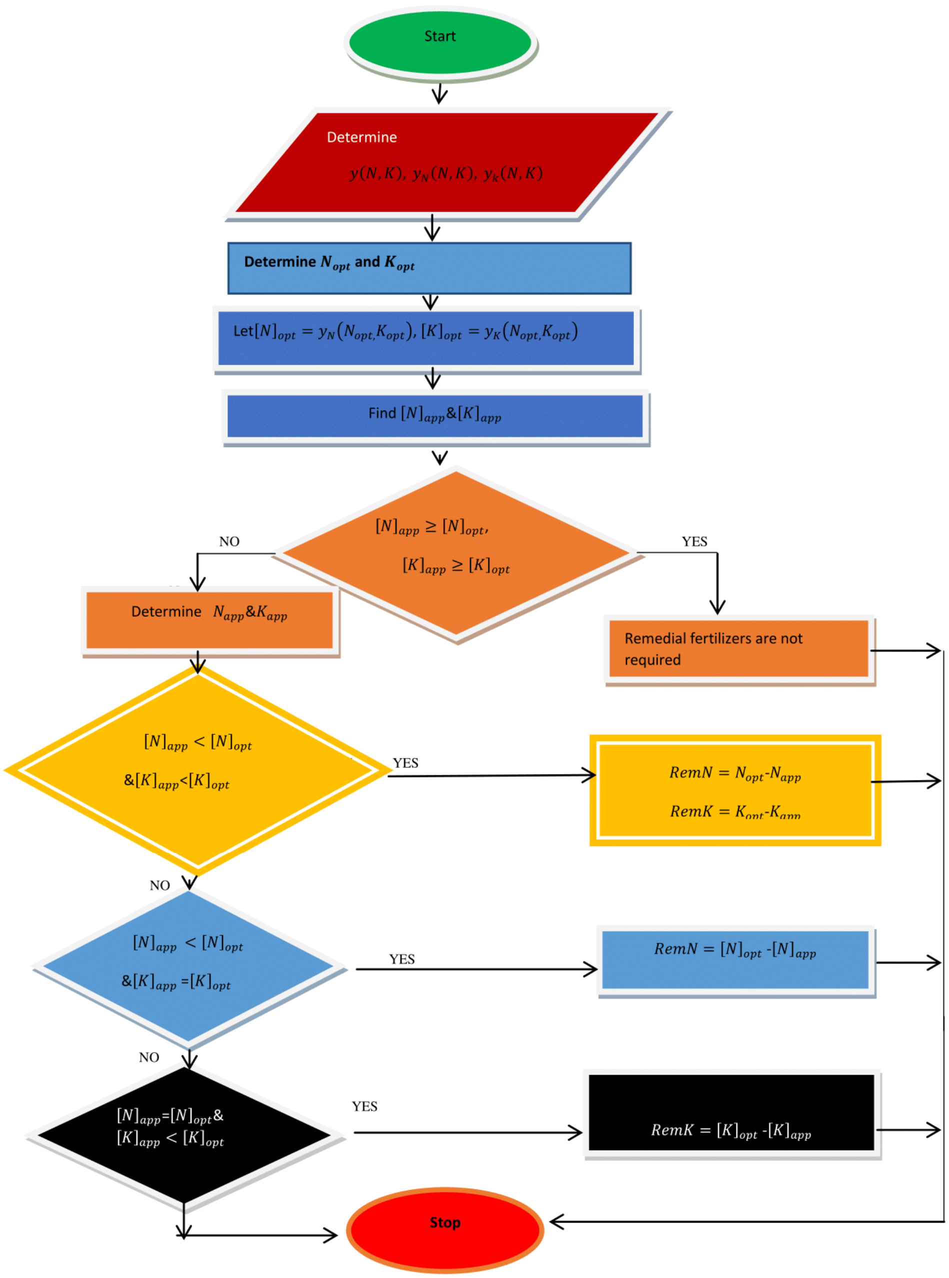

Fig.1. Algorithm of computational steps ($RemN$: Remedial fertilizer of N , $RemK$: Remedial fertilizer of K)

3. RESULTS AND DISCUSSION

We now discus how we can compute the remedial doses of fertilizers required in a potato plantation which is cultivated in the neighborhood of the trial plantation.

First the yield function (2.1) is optimized by Marquardt's method [9]. We find that

$N_{opt} = 231.07 \quad (kg/ha)$ and $K_{opt} = 81.41(kg/ha)$.

We now measure the uptake level of nitrogen and potassium for optimum yield in an intermediate stage of cultivation by putting the values N_{opt} and K_{opt} in the right hand side of the equations(2.2) and(2.3). Thus, we obtain that

$[N]_{opt} = 225.10(kg/ha)$ and $[K]_{opt} = 217.93(kg/ha)$.

It follows from the above $[N]_{opt}$ and $[K]_{opt}$ values that the potato plants having plant nutrient uptake $[N] < 225.10(kg/$ ha) and $[K] < 217.93(kg/ha)$ in the intermediate stage of cultivation can't be expected to produce maximum yield.

Let's assume that we have a case mentioned in Step-5(ii) of section-3 for the commercial potato plantation. Let us assume that $[N]_{app}$ = 100(kg/ha) and $[K]_{app}$=150(kg/ha). As discussed earlier, it can be concluded that not enough fertilizers have been used in the past and the yield is not likely to be optimum .In what follows we describe the procedure for obtaining remedial fertilizer doses that will be prescribed to farmers in order that yield can be optimized .

Step-(i) We put the values of $[N]_{app}$ = 100(kg/ha) and $[K]_{app}$ =150(kg/ha) in left hand side of equation (2.2) and (2.3) to obtain a system of two nonlinear equations as follows

$$1.867594N + 1.764794K - 0.00922N^2 - 0.00336NK - 0.00438K^2 + 1.9e-05N^3 = 100 \quad (4.1)$$

and

$$114.8226 + 0.227148N + 0.679077K + 0.000665NK - 0.00259K^2 = 150 \quad (4.2)$$

Step-(ii) We Solve the system of equations(4.1) and $(4.2)$by Newton's method [10]. The solution obtained is

N_{app}=133.18(kg/ha) and K_{app}=74.56(kg/ha)

Step-(iii) We conclude that the remedies fertilizers prescription for optimum yield in the commercial plantation are
N_{opt}-N_{app}=97.89(kg/ha) of fertilizer N,
K_{opt}-K_{app} =6.85(kg/ha) of fertilizer K.

4. CONCLUSION

We have presented here a mathematical framework by means of which crop yield can be maximized by effective application of fertilizers in a commercial plantation. The experimental results from a trial fertilization in the vicinity of commercial plantation are used for the purpose. Since the soil and environmental conditions of the trial plantation site is similar to that of commercial plantation, the findings in trial plantation can be applied to commercial plantation. The model detects shortfall of fertilizers available to the plant in an intermediate stage of cultivation, and then recommends necessary remedial fertilizer doses to be used in the commercial plantation if it is found that the fertilizers available to the commercial plantation is not adequate for optimum yield.The proposed mathematical framework can be applied in collaboration with KVKs (Krishi Vigyana Kendras) where factorial fertilizer trials can be conducted and farmers in the neighborhoods of KVKs can be advised regarding additional doses of fertilizers required in their fields in an intermediate stage of cultivation. Although we have considered potato cultivation as an example, the model presented by us can be easily extended to any commercial plantation. Here we have considered effect of Nitrogen and Potassium fertilizer on yield. The model can be extended easily to cases where other fertilizers such as phosphorus are used. In the present model, we have proposed to test the nutrients uptake of the plant once during span of cultivation. We shall obtain a more accurate prediction about remedial fertilizers requirement if plant intake is measured at various time intervals. We have not taken the economical aspect of the fertilizer use. It may so happen that the objective of optimization of yield will conflict with the objective of economic use of fertilizer. In that case we shall have to solve a multi objective optimization problem instead of a single objective optimization problem that we have solved here.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Rajanna KM. Effect of different level of nitrogen, phosphorous and potassium on growth, yield, uptake, quality and economics of potato (*Solanum tuberosum* L.), university of Agricultural science, Bangalore, Krishikosh; 1987.
2. Gomez KA, Gomez A. Statistical procedure for agricultural research. Second Edition. John Willy and Sons, New York. 1984;680.
3. Kolbe H, Wei-LI Zhang, Ballüer L. Simulation of Fertilizer Influence on Yield and Quality of Potato (*Solanum tuberosum*) Tubers by Nonlinear Optimization Method."<i>Plant and Soil</i> 1990;124(2):309-13.
4. Koech JK, Mutiso MJ, Koskei JK. Response Surface Methodology Approach to the Optimization of Potato (*Solanum tuberosum*) Tuber Yield Using Second-Order Rotatable Design, Department of Mathematics and Computer Science, University of Eldoret, Kenya Department of Statistics and Computer Science, Moi University, Eldoret, Kenya. Koech, et al., J Biom Biostat. 2017;8:3
5. Piekutowska M, Niedbała G, Piskier T, Lenartowicz T, Pilarski K, Wojciechowski T, et al. The Application of Multiple Linear Regression and Artificial Neural Network Models for Yield Prediction of Very Early Potato Cultivars before Harvest. Agronomy. 2021;11:885. Available:https://doi.org/10.3390/agronomy
6. Jackson ML. Soil chemical Analysis, Hall India Pvt. Ltd., New Delhi; 1973.
7. Singh SK, Lal SS. Effect of potassium nutrition on potato yield,q uality and nutrient use efficiency under varied levels of Nitrogen Application. 2012;39(2):155-165.
8. Ali J, Singh SP. Response of potato to nitrogen and potassium in the alluvial soil of South-Western plain zone of Uttar Pradesh. Annual of Agriculture Research New Series. 2012;33(1&2):40-44.
9. Mohan C Joshi, Kannan M Moudgalya. Optimization Theory and Practice, Narosa Publications, 4th Reprint; 2011.
10. Richard L Burden, Dougalas Faires J. Numerical analysis, 9th ed., Youngs town star university.

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